GRAND RIVER BASIN WATER MANAGEMENT STUDY
GRAND RIVER STUDY TEAM

FREQUENCY ANALYSIS OF FLOOD FLOWS IN THE GRAND RIVER BASIN

TECHNICAL REPORT #35A

PREPARED FOR THE GRAND RIVER IMPLEMENTATION COMMITTEE BY:

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FOREWORD

This report is one of a series of technical documents prepared for the Grand River Basin Water Management Study. The project described herein was undertaken by the Grand River Hydrology Subcommittee at the request of the Grand River Implementation Committee.

The material contained in this report is primarily technical support information and in itself does not necessarily constitute policy or management practices. Interpretation and evaluation of the data and findings in most cases, cannot be based solely on this report, but should be analysed in light of other reports produced within the comprehensive framework of the overall study. Questions with respect to the contents of this report should be directed to the Co-ordinator of te Grand River Water Management Study, Water Resources Branch, Ministry of the Environment, 135 St.Clair Avenue West, Toronto.

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1.0 BACKGROUND

Technical Report No. 35 is one of a series of five technical reports dealing with flood flows and flood damages in the Grand River basin. These reports are:

Technical Report	No. 35	Flow Frequency Analysis
	No. 36	Design Floods
	No. 37	Application Of The HEC-5 Model
	No. 38	Reservoir Operations
	No. 39	Flood Damages

The overall objective of the flood flows study is to produce stage-flow-frequency-damage relationships at all flood prone urban centres:

a) under natural conditions;
b) under regulation by the existing reservoir system, and as affected by existing dyke systems;
c) under regulation by proposed reservoirs, and as affected by proposed dyke and channel improvement schemes.

In order to satisfy this objective, several steps were undertaken:

In Technical Report No. 35 (this report):

1) the natural flood hydrograph was estimated for maximum annual spring freshet events at long-term gauge stations where flows are regulated by the existing reservoir system;

2) the flow-frequency relationship was established at:

   a) the long-term stations using the natural peak flows established in step 1.

   b) at long-term stations where flows have not been regulated.

In Technical Report No. 36:

3) the hydrological processes of the watershed were modelled; the 5, 10, 25, 50 and 100-year flood flows at all flood prone urban centres were obtained by simulating flood events whose peaks at stations with long-term record matched the natural flow-frequency relationship established in step 2.

In Technical Report No. 38:

4) the operation of the major reservoirs during high flow events was modelled; the effects of the reservoir systems, existing and proposed, on the 5, 10, 25, 50, and 100-year and the regional storm flood events were determined.

In Technical Report No. 39:

5) the frequency-damage relationships were estimated at each flood prone urban area for existing conditions and with

proposed dyking, channel improvement, and flood proofing schemes in place. From these relationships, average annual damages were calculated.

While the study was undertaken with the overall objective in mind, the results of the flood frequency analysis have potentially wide application and a separate reporting of the results is justified.

The purpose of this report is to present the results of the flood frequency analysis for gauge stations of long term record within the grand river basin.

2.0 Sources Of Data

2.1 Streamflow Data

Several streamflow gauges have been in operation in the Grand River basin by the Grand River Conservation Authority and/or Water Survey of Canada for at least thirty years. Information on these gauges is summarized in Table 2-1. The locations of the gauges are shown in Figure 2-1.

Table 2-1 - Long Term Streamflow Gauges In The Grand River Basin

		Period Of Record		
WSC Number	Location	Daily	Continuous	Reg. Of Flow
02GA016	Grand River below Shand Dam		1950-	Shand dam after 1942
02GA003	Grand River at Galt	1913-29	1930	Shand dam after 1942; Conestogo Dam after 1957; Guelph Dam after 1975
02GB001	Grand River at Brantford		1948-	Shand Dam after 1942; conestogo dam after 1957; Guelph Dam after 1975.
02GA015	Speed River below Guelph	1950-60	1961-	Guelph Dam after 1975
02GA018	Nith River at New Hamburg	1950-64	1965-	
02GA010	Nith River at Canning		1947-	

High flows in the Grand River basin are normally generated by one of three types of storms: spring rain on snow, summer thunderstorms, or late summer topical storms. The most frequent floods are caused by

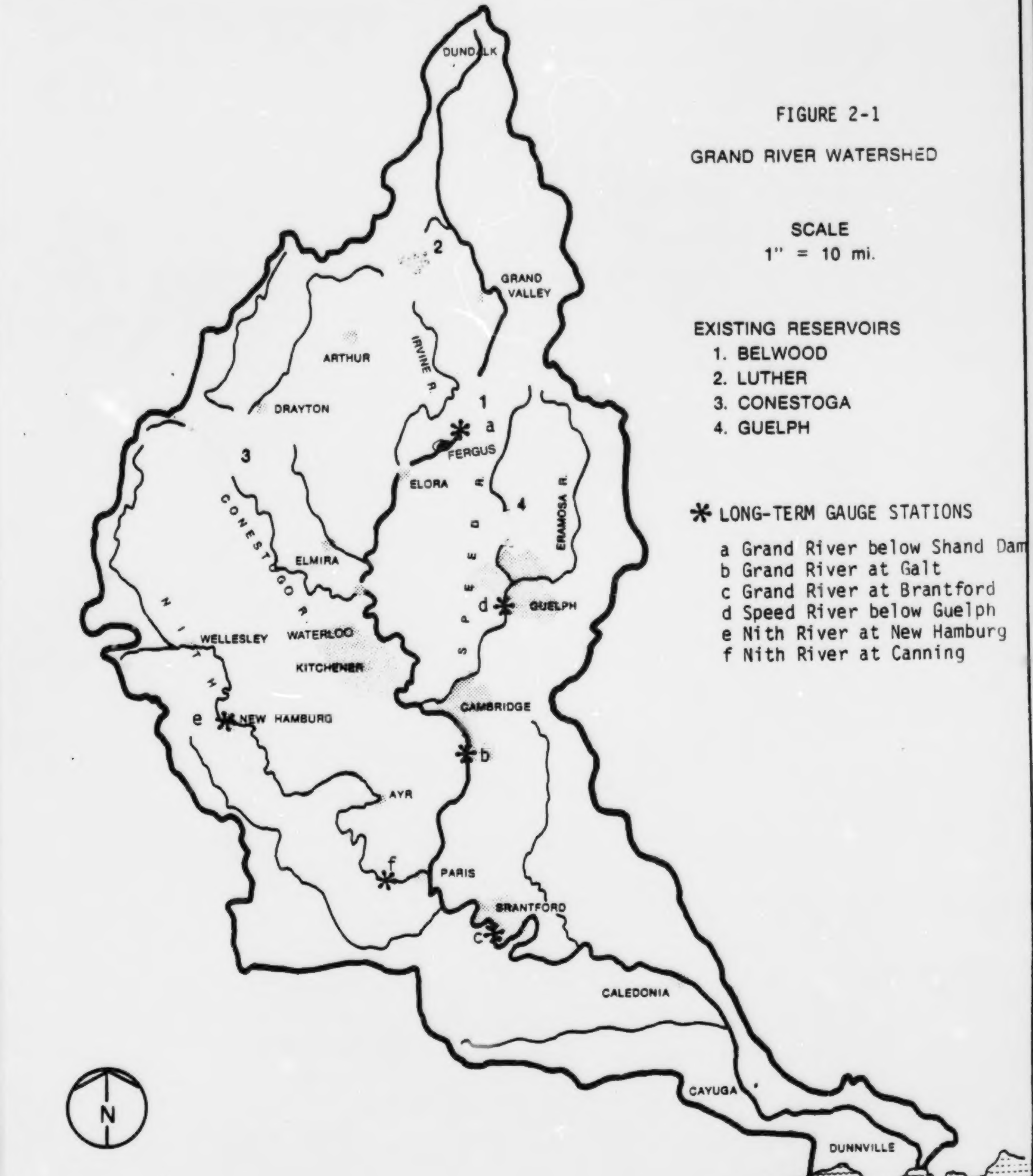


FIGURE 2-1

GRAND RIVER WATERSHED

SCALE
1" = 10 mi.

EXISTING RESERVOIRS
1. BELWOOD
2. LUTHER
3. CONESTOGA
4. GUELPH

✱ LONG-TERM GAUGE STATIONS

a Grand River below Shand Dam
b Grand River at Galt
c Grand River at Brantford
d Speed River below Guelph
e Nith River at New Hamburg
f Nith River at Canning

DUNDALK
2
GRAND VALLEY
ARTHUR
IRVINE R.
1
DRAYTON
a
FERGUS
3
ELORA
4
CONESTOGO R.
SPEED R.
ERAMOSA R.
ELMIRA
d
GUELPH
NITH R.
WELLESLEY
WATERLOO
KITCHENER
CAMBRIDGE
e
NEW HAMBURG
b
AYR
f
PARIS
BRANTFORD
c
CALEDONIA
CAYUGA
DUNNVILLE
N

spring rain on snow. Thunderstorms are not frequently wide enough in areal extent to cause an annual maximum flood on the Grand River or its major tributaries. Tropical storms do not frequently have a major effect on Southern Ontario. In order to carry out frequency analysis on a homogeneous sample, the annual spring maximum flows (those occurring in February through May) were first considerd. This sample precludes the use of the flood event caused by Hurricane Hazel. The effect of including Hurricane Hazel in the sample was also investigated (see Appendix F).

2.2 Effect Of Reservoirs

Three major reservoirs have been constructed in the last forty years for the purposes of flood control and summer flow augmentation: Shand Dam and Belwood Lake (1942), Conestogo Dam and Lake (1957), and Guelph Dam and Lake (1975). These reservoirs, having a maximum flood storage (in February) of 120,000 acre-feet, have a significant controlling effect on the flood flows on the Grand, Conestogo, and Speed Rivers. The locations of the reservoirs are shown in Figure 2-1.

Several aspects of the flow regulation contribute to non-homogenity in the sample of annual flood flows:

a) available flood storage has increased as reservoirs were constructed.

b) flood storage initially available for each event varies according to the weather conditions for months preceding the event.

c) reservoir operating strategies have change over the years as experience and access to real-time data have increased.

In order to carry out the frequency analysis of a homogeneous sample, it was thought important to estimate the natural flood flows (i.e. the flows which would have occurred had the reservoirs not been in place). The effects of the reservoirs could then be imposed in a consistent manner with the use of a reservoir operating model. The estimation of natural flows and the frequency analysis of natural flows will be discussed in this report. The effects of the reservoirs will be shown. The assumptions made and the modelling techniques used in determining the effects of the reservoirs will be discussed in Technical Report No. 38, "Reservoir Operations".

2.3 Estimating Natural Flows

The event which produces the annual maximum flow under natural conditions is not necessarily the same one that produces the observed maximum. Nor does a single event produce the maximum annual flow at each location of interest. The events that produced the annual maximum under natural conditions were selected by inspection of the natural daily flows (See Tech. Report No. 34).

For each event selected, the following data were collected:

a) hourly streamflow tables or stage charts for the Grand River at Galt and Brantford from Water Survey of Canada (WSC) or the Grand River Conservation Authority (GRCA);

b) hourly streamflow tables or stage charts for the Grand River below Shand Dam from WSC or GRCA, or operators' logs on gate and valve openings at the Shand Dam from GRCA;
c) operators' logs on gate and valve openings at the Conestogo Dam from GRCA;
d) lake level recording charts or operators' logs on lake levels at Shand and Conestogo Dams from GRCA;
e) stage-storage and stage-discharge curves from Shand and Conestogo Dams from GRCA.

The reservoir inflows were calculated based on the hydrologic continuity equation as follows:

inflow = discharge + (change in lake storage)

However, to produce a smooth hourly hydrograph from input data at irregular intervals, the inflows were calculated by:

a) attributing the average inflow to the midpoint of the interval.
b) connecting these midpoints when trend is continuing in one direction.
c) connecting an extrapolation from each midpoint when trend changes during the interval.

The volume of inflow is strictly preserved.

The natural flow hydrograph for each event at Galt was estimated by replacing the portion attributed to reservoir discharge with the inflow. The steps involved were as follows:

a) reservoir discharges from Shand and, if applicable, Conestogo Dams were routed, using the Muskingum routing technique, to Galt.
b) the routed discharge hydrographs were subtracted from the observed hydrograph at Galt to produce a new hydrograph of local flows.
c) the reservoir inflow hydrographs were routed to Galt and added to the local hydrograph.

Muskingum K and X equal to 17 hours and 0.4 were used to route from both Shand Dam and Conestogo Dams to Galt. The values of these parameters were determined based on recent high flow events.

The natural flow hydrogrpah for each event at Brantford was estimated in a similar manner by replacing the portion attributed to the observed flow from Galt by the natural flow at Galt. Muskingum K and X equal to 5 hours and 0.4 were used.

Since 1974, natural hydrographs have been produced at all of these stations by the real-time flood forecasting model used by GRCA. The procedure is the same; however, streamflow data at many more gauges are included, thereby providing an even more reliable estimate. The natural flow hydrographs available by this means were used.

Much of the required data for the 1950's was not available. At that time, daily data was more generally used so operator's logs were not as complete. Some recording charts for major events were lost during follow-up studies. The hourly flow hydrographs could not be estimated using the above procedures.

TABLE 2-2 - ANNUAL MAXIMUM INSTANTANEOUS NATURAL FLOWS

Year	SHAND DAM		CONESTOGO DAM		GALT		BRANTFORD		GUELPH		NEW HAMBURG		CANNING	
	Date	Flow	Date	Flow	Date	Flow	Date	Flow	Date	Flow	Date	Flow	Date	Flow
1948	*Mar 21*	*25800*	*Mar 20*	*12600*	*Mar 20*	*57600*	*Mar 20*	*61100*	------	------	------	------	Mar 21	14900
1949	*Mar 28*	*7870*	*Mar 23*	*7900*	*Mar 23*	*30100*	*Mar 23*	*29800*	------	------	------	------	Mar 23	29800
1950	*Apr 5*	*12400*	*Apr 4*	*9410*	*Apr 5*	*35900*	*Apr 5*	*53100*	Apr 5	6720	------	------	Apr 5	12600
1951	*Mar 31*	*10200*	*Mar 30*	*7110*	*Mar 31*	*30300*	*Mar 31*	*36200*	Mar 1	3780	Mar 31	5570	Apr 1	6720
1952	*Apr 2*	*8520*	*Apr 1*	*6750*	*Apr 2*	*21400*	*Apr 2*	*25500*	*Apr 1*	*1870*	Mar 22	3630	Mar 22	3970
1953	*May 2*	*4730*	*Mar 16*	*4630*	*Mar 16*	*13600*	*May 3*	*18600*	*Mar 4*	*1660*	*May 26*	*3630*	Mar 17	4590
1954	*Mar 2*	*9900*	*Feb 16*	*9890*	*Apr 17*	*23900*	*Mar 2*	*38300*	*Apr 17*	*2870*	*Mar 2*	*6270*	Feb 17	12900
1955	Apr 3	11700	*Apr 2*	*4960*	Mar 12	17230	Mar 12	26000	*Mar 11*	*2420*	*Apr 2*	*4530*	Mar 12	8940
1956	Apr 4	13300	*Apr 4*	*9650*	Apr 5	31100	Apr 5	40500	*Apr 5*	*3670*	Apr 4	7650	Apr 5	9420
1957	*Feb 27*	*4490*	*Feb 26*	*4620*	*Feb 27*	*12200*	*Feb 27*	*20300*	*Apr 6*	*1930*	*Feb 26*	*4060*	Feb 27	4850
1958	*Mar 31*	*3280*	*Mar 29*	*3500*	*Mar 29*	*7210*	*Mar 29*	*11300*	*Apr 7*	*712*	*Mar 28*	*3170*	Mar 28	2360
1959	Apr 8	8050	Apr 5	6350	Apr 6	32900	Apr 6	35700	*Apr 6*	*3420*	Apr 6	5990	Apr 3	6510
1960	Apr 3	11200	Apr 3	13000	Apr 4	46300	Apr 4	55500	Apr 4	3010	Apr 4	8820	Apr 5	10200
1961	Mar 28	3460	Feb 25	3650	Feb 25	10400	Feb 25	14100	*Feb 24*	*1910*	Feb 25	2470	Feb 23	6500
1962	Mar 30	10400	Mar 30	12800	Mar 30	29700	Mar 30	34600	Mar 30	2170	Mar 30	7010	Mar 31	7320
1963	Mar 27	9680	Mar 28	11700	Mar 28	28200	Mar 28	37000	Mar 27	3000	Mar 26	9220	Mar 37	10700
1964	Apr 8	4530	Apr 8	8340	Apr 8	15000	Apr 8	17000	Apr 8	1020	*Apr 7*	*3750*	*Apr 8*	*3610*
1965	Apr 12	5980	Feb 10	15600	Apr 8	26700	Apr 8	33600	Apr 8	3460	Feb 11	9540	Feb 11	12500
1966	Mar 19	3220	Mar 18	3130	Mar 19	9380	Mar 19	9320	Mar 19	920	*Feb 12*	*2230*	*Feb 12*	*3540*
1967	Apr 3	8510	Apr 3	12000	Apr 3	37000	Apr 4	36500	Apr 3	4810	Apr 3	11800	Apr 4	10400
1968	Mar 28	5580	Mar 21	7440	Mar 21	14600	Mar 21	18400	Mar 20	2300	Feb 2	9880	*Feb 3*	*7040*
1969	Apr 18	5710	Apr 18	8280	Apr 19	23800	Apr 19	24300	Apr 5	2750	Apr 5	6950	Apr 6	7010
1970	Apr 14	8300	Apr 14	6720	Apr 15	17900	Apr 15	17600	Apr 9	1640	Apr 9	4440	Apr 10	5100
1971	Apr 12	8290	Apr 12	5250	Apr 13	21400	Apr 12	24300	Apr 10	1700	Apr 10	3680	Apr 4	4280
1972	Apr 18	14100	Apr 17	8590	Apr 19	32400	Apr 19	37900	Apr 15	3730	Apr 13	6290	Apr 14	7350
1973	Mar 12	7130	Mar 11	8990	Mar 12	28200	Mar 12	33800	Mar 12	2930	Mar 12	6800	Mar 13	7160
1974	Mar 7	14100	May 17	16650	May 17	55200	May 17	64700	May 17	5850	Mar 5	11000	Mar 6	9710
1975	Apr 19	15600	Apr 19	20800	Apr 19	58700	Apr 20	64000	Apr 19	4480	Apr 19	16000	Apr 20	14800
1976	Mar 21	14100	Mar 21	17200	Mar 22	42800	Mar 22	52200	Mar 21	3500	Mar 21	11400	Mar 22	11100
1977	Mar 14	13100	Mar 13	15320	Mar 14	40400	Mar 14	53200	Mar 14	3700	Mar 13	12800	Mar 14	12800
1978	Apr 12	8120	Apr 11	8460	Apr 12	26600	Apr 12	33100	Apr 12	2280	Apr 11	7030	Apr 12	7900
1979	Apr 14	12500	Apr 14	13600	Apr 14	36000	Apr 15	52700	Apr 14	3340	Apr 14	13700	Apr 15	13500
1954					Oct 16	59800	Oct 17	64700	Oct 16	5740	Oct 16	12000	Oct 17	15100

Flows printed in italics are estimated based on daily mean flows.

In order to obtain a continuous sample of annual maximum instantaneous flows, correlation equations were developed between maximum instantaneous and corresponding daily mean flows. The correlation equations and the plots of maximum instantaneous versus maximum daily flows are shown in Appendix A.

The maximum instantaneous natural flows are summarized in Table 2-2.

THe maximum instantaneous natural flows versus the corresponding observed flows are shown in Appendix B.

3. FREQUENCY DISTRIBUTIONS AND ANALYSIS

The frequency analysis was prepared using the Flood Damage Reduction Program Flood Frequency Analysis program (FDRPFFA) developed by Condie, Nix, and Boone, Inland Water Directorate, Environment Canada 1977.

The program gives the flood frequency regime as obtained using the Gumbel I, the Log-Normal, the Three Parameter Log-Normal, and the Log-Pearson Type III distributions using the maximum likelihood method of fitting. A moment fit is used when the maximum likelihood estimators do not have true solutions and is used for the Log-Pearson Type III distribution in addition to the maximum likelihood method. The asymptotic standard error of estimate is given as a percentage for selected return periods.

The program was written in FORTRAN IV. For use on the IBM 5100 desktop computer, it was converted to APL.

A description of all distributions and a discussion of characteristics to be considered to determine reasonableness of fit are included in Appendix C.

3.1 Flood Frequency Analysis

Tables 3-1 to 3-14 show the flood frequency analysis using the four frequency distributions, as follows:

Tables	
3-1 and 3-2	Natural Flow-Grand River at Shand
3-3 and 3-4	Natural Flow-Conestogo River at Coenstogo Dam
3-5 and 3-6	Natural Flow-Grand River at Galt
3-7 and 3-8	Natural Flow-Grand River at Brantford
3-9 and 3-10	Natural Flow-Speed River below Guelph
3-11 and 3-12	Nith River at New Hamburg
3-13 and 3-14	Nith River at Canning

3.2 Selection of Flood Frequency Distribution

In order to compare the fit of the Gumbel I, Log-Normal, and Three parameter Log-Normal distributions, the coefficients of skew and kurtosis of the sample were compared with the theoretical values of these coefficients. In the case of the Gumbel I distribution, the coefficients of skew and kurtosis of the sample should approach 1.14 and 5.4 respectively. In the case of the Log-Normal distribution, the

Table 3-1. Data Base and Plotting Positions

NATURAL FLOW - GRAND RIVER AT SHAND DAM
MAXIMUM INSTANTANEOUS FLOW (CUBIC FEET PER SECOND)
1948 - 1979

YEAR	DATA	ORDERED	RANK	PROB.	RET. PERIOD
1948	25800	25800	1	.030	33.000
1949	7870	15600	2	.061	16.500
1950	12400	14100	3	.091	11.000
1951	10200	14100	4	.121	8.250
1952	8520	14100	5	.152	6.600
1953	4730	13300	6	.182	5.500
1954	9900	13100	7	.212	4.714
1955	11700	12500	8	.242	4.125
1956	13300	12400	9	.273	3.667
1957	4490	11700	10	.303	3.300
1958	3280	11200	11	.333	3.000
1959	8050	10400	12	.364	2.750
1960	11200	10200	13	.394	2.538
1961	3460	9900	14	.424	2.357
1962	10400	9680	15	.455	2.200
1963	9680	8520	16	.485	2.063
1964	4530	8510	17	.515	1.941
1965	5980	8300	18	.545	1.833
1966	3220	8290	19	.576	1.737
1967	8510	8120	20	.606	1.650
1968	5580	8050	21	.636	1.571
1969	5710	7870	22	.667	1.500
1970	8300	7130	23	.697	1.435
1971	8290	5980	24	.727	1.375
1972	14100	5710	25	.758	1.320
1973	7130	5580	26	.788	1.269
1974	14100	4730	27	.818	1.222
1975	15600	4530	28	.848	1.179
1976	14100	4490	29	.879	1.138
1977	13100	3460	30	.909	1.100
1978	8120	3280	31	.939	1.065
1979	12500	3220	32	.970	1.031

Table 3-2. Flood Estimates

```
          NATURAL FLOW - GRAND RIVER AT SHAND DAM
    MAXIMUM INSTANTANEOUS FLOW (CUBIC FEET PER SECOND)
                       1948 - 1979

          SAMPLE STATISTICS

      MEAN =     9495          S.D. =    4637.9          C.S. =    1.3024          C.K. =    6.8325

          SAMPLE STATISTICS (LOGS)

      MEAN =   9.0446          S.D. =     .4967          C.S. =    ¯.2784          C.K. =    3.1229

      SAMPLE MIN =    3220     SAMPLE MAX =   25800     N =  32

      PARAMETERS FOR GUMBEL I     A =  .000283     U =        7450

      PARAMETERS FOR LOGNORMAL     M =  9.0446     S =  .4967

      PARAMETERS FOR THREE PARAMETER LOGNORMAL     A =     ¯1824     M =  9.2575     S =  .3998

          STATISTICS OF LOG(X-A)

      MEAN =   9.2575          S.D. =     .3998          C.S. =    ¯.0579          C.K. =    3.1979

      PARAMETERS FOR LOG PEARSON III BY MOMENTS     A= ¯.0691 B=   51.6098 LOG(M)=12.6126     M =      300311

      PARAMETERS FOR LOG PEARSON III BY MAXIMUM LIKELIHOOD     A = ¯.0742     B =   43.4179   LOG(M) =12.2673    M =       212632

      DISTRIBUTION STATISTICS     MEAN =  9.0446     S.D =  .4891     C.S. = ¯.3035




             GUMBEL I            LOGNORMAL        THREE PARAMETER             LOG PEARSON III
                                                     LOGNORMAL        MAXIMUM LIKELIHOOD          MOMENTS
   RETURN     FLOOD   ST.ERROR  FLOOD    ST. ERROR  FLOOD    ST. ERROR  FLOOD    ST. ERROR  FLOOD    ST. ERROR
   PERIOD   ESTIMATE  PERCENT  ESTIMATE  PERCENT  ESTIMATE  PERCENT   ESTIMATE  PERCENT   ESTIMATE  PERCENT

    1.005     1560              2890                2590                2680                2660
    1.050     3520              3740                3610                3640                3610
    1.250     5770              5580                5660                5660                5620
    2.000     8740              8470                8660                8680                8670
    5.000    12700     8.82    12900     10.20     12900      9.39     12900      9.27     12900     15.60
   10.000    15400     9.37    16000     11.90     15700     10.90     15600     10.30     15700     19.30
   20.000    17900     9.84    19200     13.50     18400     13.00     18100     12.00     18400     24.50
   50.000    21200    10.30    23500     15.50     22000     16.00     21300     15.30     21800     32.70
  100.000    23700    10.60    26900     16.90     24700     18.30     23700     18.20     24300     39.70
  200.000    26100    10.90    30500     18.20     27500     20.60     26000     21.20     26800     47.10
  500.000    29400    11.20    35400     19.90     31300     23.60     29000     25.50     30000     57.30
```

Table 3-3. Data Base and Plotting Positions

NATURAL FLOW - CONESTOGO RIVER AT CONESTOGO DAM
MAXIMUM INSTANTANEOUS FLOW (CUBIC FEET PER SECOND)
~~1948 - 1979~~

YEAR	DATA	ORDERED	RANK	PROB.	RET. PERIOD
1948	12600	20800	1	.030	33.000
1949	7900	17200	2	.061	16.500
~~1950~~	~~9410~~	~~16550~~	~~3~~	~~.091~~	~~11.000~~
1951	7110	15600	4	.121	8.250
1952	6750	15320	5	.152	6.600
~~1953~~	~~4630~~	~~13600~~	~~6~~	~~.182~~	~~5.500~~
1954	9890	13000	7	.212	4.714
1955	4960	12800	8	.242	4.125
~~1956~~	~~9650~~	~~12600~~	~~9~~	~~.273~~	~~3.667~~
1957	4620	12000	10	.303	3.300
1958	3500	11700	11	.333	3.000
~~1959~~	~~6350~~	~~9890~~	~~12~~	~~.364~~	~~2.750~~
1960	13000	9650	13	.394	2.538
1961	3650	9410	14	.424	2.357
~~1962~~	~~12800~~	~~8990~~	~~15~~	~~.455~~	~~2.200~~
1963	11700	8590	16	.485	2.063
1964	8340	8460	17	.515	1.941
~~1965~~	~~15600~~	~~8340~~	~~18~~	~~.545~~	~~1.833~~
1966	3130	8280	19	.576	1.737
1967	12000	7900	20	.606	1.650
~~1968~~	~~7440~~	~~7440~~	~~21~~	~~.636~~	~~1.571~~
1969	8280	7110	22	.667	1.500
1970	6720	6750	23	.697	1.435
~~1971~~	~~5250~~	~~6720~~	~~24~~	~~.727~~	~~1.375~~
1972	8590	6350	25	.758	1.320
1973	8990	5250	26	.788	1.269
~~1974~~	~~16550~~	~~4960~~	~~27~~	~~.818~~	~~1.222~~
1975	20800	4630	28	.848	1.179
1976	17200	4620	29	.879	1.138
~~1977~~	~~15320~~	~~3650~~	~~30~~	~~.909~~	~~1.100~~
1978	8460	3500	31	.939	1.065
1979	13600	3130	32	.970	1.031

Table 3-4. Flood Estimates

```
     NATURAL FLOW - CONESTOGO RIVER AT CONESTOGO DAM
   MAXIMUM INSTANTANEOUS FLOW (CUBIC FEET PER SECOND)
                   1948 - 1979

          SAMPLE STATISTICS

   MEAN =       9525          S.D. =     4429.8          C.S. =        .6534          C.K. =     3.2168

          SAMPLE STATISTICS (LOGS)

   MEAN =     9.0513          S.D. =      .4908          C.S. =       ¯.2831          C.K. =     2.7446

   SAMPLE MIN =     3130      SAMPLE MAX =    20800      N =  32

   PARAMETERS FOR GUMBEL I      A =  .000285      U =       7484

   PARAMETERS FOR LOGNORMAL      M =  9.0513      S =  .4908

   PARAMETERS FOR THREE PARAMETER LOGNORMAL      A =     ¯1773      M =  9.2573      S =  .3982

          STATISTICS OF LOG(X=A)

   MEAN =     9.2573          S.D. =      .3982          C.S. =       ¯.1082          C.K. =     2.6301

   PARAMETERS FOR LOG PEARSON III BY MOMENTS      A= ¯.0695 B=    49.9041 LOG(M)=12.5183      M =       273300

   PARAMETERS FOR LOG PEARSON III BY MAXIMUM LIKELIHOOD      A = ¯.1387      B =    12.4843   LOG(M) =10.7826    M =        48176

   DISTRIBUTION STATISTICS      MEAN =  9.0513      S.D =  .4900      C.S. = ¯.5660
```

	GUMBEL I		LOGNORMAL		THREE PARAMETER LOGNORMAL		LOG PEARSON III MAXIMUM LIKELIHOOD		LOG PEARSON III MOMENTS	
RETURN PERIOD	FLOOD ESTIMATE	ST.ERROR PERCENT	FLOOD ESTIMATE	ST. ERROR PERCENT	FLOOD ESTIMATE	ST. ERROR PERCENT	FLOOD ESTIMATE	ST. ERROR PERCENT	FLOOD ESTIMATE	ST. ERROR PERCENT
1.005	1630		2950		2650		2500		2710	
1.050	3580		3800		3670		3550		3660	
1.250	5010		5640		5720		5760		5690	
2.000	8770		8530		8710		8930		8730	
5.000	12800	8.77	12900	10.10	12900	9.32	13000	8.51	13000	15.60
10.000	15400	9.33	16000	11.70	15700	10.90	15400	8.86	15700	19.20
20.000	17900	9.80	19100	13.30	18400	12.90	17500	9.88	18300	24.40
50.000	21200	10.30	23300	15.30	21900	15.90	20000	12.00	21600	32.60
100.000	23600	10.60	26700	16.70	24700	18.20	21700	14.00	24100	39.60
200.000	26100	10.90	30200	18.00	27500	20.50	23300	16.20	26500	47.00
500.000	29300	11.20	35100	19.70	31200	23.40	25300	19.30	29700	57.20

Table 3-5. Data Base and Plotting Positions

NATURAL FLOW - GRAND RIVER AT GALT
MAXIMUM INSTANTANEOUS FLOW (CUBIC FEET PER SECOND)
1948 - 1979

YEAR	DATA	ORDERED	RANK	PROB.	RET. PERIOD
1948	57600	58700	1	.030	33.000
1949	30100	57600	2	.061	16.500
1950	35900	55200	3	.091	11.000
1951	30300	46300	4	.121	8.250
1952	21400	42800	5	.152	6.600
1953	13600	40400	6	.182	5.500
1954	23900	37000	7	.212	4.714
1955	17230	36000	8	.242	4.125
1956	31100	35900	9	.273	3.667
1957	12200	32900	10	.303	3.300
1958	7210	32400	11	.333	3.000
1959	32900	31100	12	.364	2.750
1960	46300	30300	13	.394	2.538
1961	10400	30100	14	.424	2.357
1962	29700	29700	15	.455	2.200
1963	28200	28200	16	.485	2.063
1964	15000	28200	17	.515	1.941
1965	26700	26700	18	.545	1.833
1966	9380	26600	19	.576	1.737
1967	37000	23900	20	.606	1.650
1968	14600	23800	21	.636	1.571
1969	23800	21400	22	.667	1.500
1970	17900	21400	23	.697	1.435
1971	21400	17900	24	.727	1.375
1972	32400	17230	25	.758	1.320
1973	28200	15000	26	.788	1.269
1974	55200	14600	27	.818	1.222
1975	58700	13600	28	.848	1.179
1976	42800	12200	29	.879	1.138
1977	40400	10400	30	.909	1.100
1978	26600	9380	31	.939	1.065
1979	36000	7210	32	.970	1.031

Table 3-6. Flood Estimates

```
          NATURAL FLOW - GRAND RIVER AT GALT
   MAXIMUM INSTANTANEOUS FLOW (CUBIC FEET PER SECOND)
                       1948 - 1979

          SAMPLE STATISTICS

    MEAN =      28566          S.D. =   13638.7          C.S. =       .5836          C.K..=     3.2720

          SAMPLE STATISTICS (LOGS)

    MEAN =    10.1365          S.D. =      .5301         C.S. =     ¯.5431          C.K. =     3.1476

    SAMPLE MIN =     7210     SAMPLE MAX =    58700      N =  32

    PARAMETERS FOR GUMBEL I       A =  .000090     U =       22224

    PARAMETERS FOR LOGNORMAL      M = 10.1365      S =  .5301

    PARAMETERS FOR THREE PARAMETER LOGNORMAL      A =    ¯16980      M = 10.6832      S =  .3004

          STATISTICS OF LOG(X-A)

    MEAN =    10.6832          S.D. =      .3004         C.S. =     ¯.0681          C.K. =     2.7912

    PARAMETERS FOR LOG PEARSON III BY MOMENTS      A= ¯.1439 B=   13.5622 LOG(M)=12.0886      M =       177825

    PARAMETERS FOR LOG PEARSON III BY MAXIMUM LIKELIHOOD      A = ¯.2201      B =     5.8757   LOG(M) =11.4295     M =          91998

    DISTRIBUTION STATISTICS      MEAN = 10.1365     S.D =  .5334     C.S. = ¯.8251


                GUMBEL I             LOGNORMAL        THREE PARAMETER              LOG PEARSON III
                                                         LOGNORMAL         MAXIMUM LIKELIHOOD           MOMENTS
  RETURN    FLOOD    ST.ERROR  FLOOD    ST. ERROR  FLOOD    ST. ERROR  FLOOD    ST. ERROR  FLOOD    ST. ERROR
  PERIOD   ESTIMATE  PERCENT  ESTIMATE  PERCENT   ESTIMATE  PERCENT   ESTIMATE  PERCENT   ESTIMATE  PERCENT

   1.005     3590               8010                5780                6150                6750
   1.050     9790              10600                9630                9450                9810
   1.250    16900              16200               16900               16700               16500
   2.000    26300              25200               26600               27100               26500
   5.000    39000     9.13     39500     10.90     39200      8.86     39800      8.36     39700     17.20
  10.000    47400     9.64     49800     12.60     47100     10.00     46900      8.18     47900     22.00
  20.000    55400    10.10     60400     14.40     54500     11.70     52800      8.51     55200     28.90
  50.000    65800    10.60     74900     16.50     63800     14.20     59200      9.64     64000     40.00
 100.000    73600    10.80     86600     18.00     70700     16.20     63300     10.90     70100     49.40
 200.000    81400    11.10     98900     19.50     77600     18.20     66900     12.40     75800     59.30
 500.000    91600    11.40    116000     21.30     86600     20.80     71000     14.50     82900     73.10
```

Table 3-7. Data Base and Plotting Positions

NATURAL FLOW - GRAND RIVER AT BRANTFORD
MAXIMUM INSTANTANEOUS FLOW (CUBIC FEET PER SECOND)
1948 - 1979

YEAR	DATA	ORDERED	RANK	PROB.	RET. PERIOD
1948	61100	64700	1	.030	33.000
1949	29800	64000	2	.061	16.500
1950	53100	61100	3	.091	11.000
1951	36200	55500	4	.121	8.250
1952	25500	53200	5	.152	6.600
1953	18600	53100	6	.182	5.500
1954	38300	52700	7	.212	4.714
1955	26000	52200	8	.242	4.125
1956	40500	40500	9	.273	3.667
1957	20300	38300	10	.303	3.300
1958	11300	37900	11	.333	3.000
1959	35700	37000	12	.364	2.750
1960	55500	36500	13	.394	2.538
1961	14100	36200	14	.424	2.357
1962	34600	35700	15	.455	2.200
1963	37000	34600	16	.485	2.063
1964	17000	33800	17	.515	1.941
1965	33600	33600	18	.545	1.833
1966	9320	33100	19	.576	1.737
1967	36500	29800	20	.606	1.650
1968	18400	26000	21	.636	1.571
1969	24300	25500	22	.667	1.500
1970	17600	24300	23	.697	1.435
1971	24300	24300	24	.727	1.375
1972	37900	20300	25	.758	1.320
1973	33800	18600	26	.788	1.269
1974	64700	18400	27	.818	1.222
1975	64000	17600	28	.848	1.179
1976	52200	17000	29	.879	1.138
1977	53200	14100	30	.909	1.100
1978	33100	11300	31	.939	1.065
1979	52700	9320	32	.970	1.031

Table 3-8. Flood Estimates

```
          NATURAL FLOW - GRAND RIVER AT BRANTFORD
    MAXIMUM INSTANTANEOUS FLOW (CUBIC FEET PER SECOND)
                     1948 - 1979

            SAMPLE STATISTICS

       MEAN =      34694         S.D. =   15685.5          C.S. =      .3565         C.K. =    2.5543

            SAMPLE STATISTICS (LOGS)

       MEAN =   10.3415          S.D. =      .5066         C.S. =     ¯.5525         C.K. =    3.0631

       SAMPLE MIN =     9320     SAMPLE MAX =    64700     N =  32

       PARAMETERS FOR GUMBEL I     A =  .000076     U =      27264

       PARAMETERS FOR LOGNORMAL     M = 10.3415     S =  .5066

       PARAMETERS FOR THREE PARAMETER LOGNORMAL     A =   ¯27183     M = 11.0015     S =  .2559

            STATISTICS OF LOG(X-A)

       MEAN =   11.0015          S.D. =      .2559         C.S. =     ¯.0764         C.K. =    2.5128

       PARAMETERS FOR LOG PEARSON III BY MOMENTS     A= ¯.1400 B=   13.1029 LOG(M)=12.1753     M =       193933

       PARAMETERS FOR LOG PEARSON III BY MAXIMUM LIKELIHOOD     A = ¯.3647     B =    2.3117  LOG(M) =11.1845   M =        72009

       DISTRIBUTION STATISTICS     MEAN = 10.3415     S.D =  .5545     C.S. =¯1.3154


              GUMBEL I             LOGNORMAL        THREE PARAMETER             LOG PEARSON III
                                                        LOGNORMAL         MAXIMUM LIKELIHOOD          MOMENTS
     RETURN    FLOOD   ST.ERROR  FLOOD    ST. ERROR  FLOOD    ST. ERROR  FLOOD    ST. ERROR  FLOOD    ST. ERROR
     PERIOD   ESTIMATE  PERCENT  ESTIMATE  PERCENT   ESTIMATE  PERCENT   ESTIMATE  PERCENT   ESTIMATE  PERCENT

      1.005     5300               10300               7260               6220               8760
      1.050    12600               13500              12200              10700              12500
      1.250    21000               20200              21200              20900              20600
      2.000    32100               31000              32800              34800              32500
      5.000    47000      8.92     47500     10.40    47200      8.31    49200      6.69    47800     16.80
     10.000    56900      9.46     59300     12.10    56100      9.30    55800      5.85    57100     21.60
     20.000    66400      9.92     71300     13.70    64200     10.80    60500      5.15    65400     28.40
     50.000    78600     10.40     87600     15.80    74200     13.20    64700      4.41    75100     39.30
    100.000    87800     10.70    101000     17.20    81600     15.00    67000      4.02    82000     48.60
    200.000    97000     11.00    114000     18.60    88800     16.90    68600      3.80    88300     58.40
    500.000   109000     11.20    133000     20.30    98100     19.30    70000      3.76    96100     72.00
```

Table 3-9. Data Base and Plotting Positions

NATURAL FLOW - SPEED RIVER BELOW GUELPH
MAXIMUM INSTANTANEOUS FLOW (CUBIC FEET PER SECOND)
1950 - 1979

YEAR	DATA	ORDERED	RANK	PROB.	RET. PERIOD
1950	6720	6720	1	.032	31.000
1951	3780	5850	2	.065	15.500
1952	1870	4810	3	.097	10.333
1953	1660	4480	4	.129	7.750
1954	2870	3780	5	.161	6.200
1955	2420	3770	6	.194	5.167
1956	3770	3730	7	.226	4.429
1957	1460	3700	8	.258	3.875
1958	712	3500	9	.290	3.444
1959	3410	3460	10	.323	3.100
1960	3010	3410	11	.355	2.818
1961	1430	3340	12	.387	2.583
1962	2170	3010	13	.419	2.385
1963	3000	3000	14	.452	2.214
1964	1020	2930	15	.484	2.067
1965	3460	2870	16	.516	1.938
1966	920	2750	17	.548	1.824
1967	4810	2420	18	.581	1.722
1968	2300	2300	19	.613	1.632
1969	2750	2280	20	.645	1.550
1970	1640	2170	21	.677	1.476
1971	1700	1870	22	.710	1.409
1972	3730	1700	23	.742	1.348
1973	2930	1660	24	.774	1.292
1974	5850	1640	25	.806	1.240
1975	4480	1460	26	.839	1.192
1976	3500	1430	27	.871	1.148
1977	3700	1020	28	.903	1.107
1978	2280	920	29	.935	1.069
1979	3340	712	30	.968	1.033

Table 3-10. Flood Estimates

```
         NATURAL FLOW - SPEED RIVER BELOW GUELPH
    MAXIMUM INSTANTANEOUS FLOW (CUBIC FEET PER SECOND)
                      1950 - 1979

          SAMPLE STATISTICS

       MEAN =       2890          S.D. =    1403.0          C.S. =      .7812          C.K. =     4.1762

          SAMPLE STATISTICS (LOGS)

       MEAN =     7.8444          S.D. =      .5326         C.S. =     ¯.5602          C.K. =     3.4246

       SAMPLE MIN =      712     SAMPLE MAX =     6720     N =  30

       PARAMETERS FOR GUMBEL I      A =  .000886      U =       2247

       PARAMETERS FOR LOGNORMAL       M =  7.8444      S =  .5326

       PARAMETERS FOR THREE PARAMETER LOGNORMAL      A =     ¯1534     M =  8.3471      S =  .3151

          STATISTICS OF LOG(X-A)

       MEAN =     8.3471          S.D. =      .3151         C.S. =     ¯.0505          C.K. =     3.0753

       PARAMETERS FOR LOG PEARSON III BY MOMENTS      A= ¯.1492 B=   12.7466 LOG(M)= 9.7460      M =         17085

       PARAMETERS FOR LOG PEARSON III BY MAXIMUM LIKELIHOOD      A = ¯.1724      B =     9.3584    LOG(M) = 9.4575     M =          12803

       DISTRIBUTION STATISTICS      MEAN =  7.8444      S.D =  .5273     C.S. = ¯.6538



                 GUMBEL I            LOGNORMAL        THREE PARAMETER             LOG PEARSON III
                                                         LOGNORMAL       MAXIMUM LIKELIHOOD          MOMENTS
     RETURN    FLOOD   ST.ERROR  FLOOD   ST. ERROR  FLOOD   ST. ERROR  FLOOD   ST. ERROR  FLOOD   ST. ERROR
     PERIOD   ESTIMATE  PERCENT ESTIMATE  PERCENT  ESTIMATE  PERCENT  ESTIMATE  PERCENT  ESTIMATE  PERCENT

      1.005      364                805                598                664                674
      1.050      990               1060                978                984                985
      1.250     1710               1630               1700               1680               1660
      2.000     2660               2550               2680               2700               2680
      5.000     3940     9.43      4000     11.30     3970      9.29     4010      9.15     4030     17.80
     10.000     4790     9.95      5050     13.10     4780     10.50     4780      9.34     4850     22.90
     20.000     5600    10.40      6130     14.90     5550     12.30     5450     10.20     5590     30.20
     50.000     6650    10.90      7610     17.10     6520     15.00     6230     12.10     6460     41.80
    100.000     7440    11.20      8810     18.70     7240     17.10     6750     14.10     7070     51.70
    200.000     8230    11.50     10100     20.20     7960     19.20     7230     16.20     7640     62.20
    500.000     9260    11.70     11800     22.10     8920     22.00     7810     19.20     8340     76.80
```

Table 3-11. Data Base and Plotting Positions

NITH RIVER AT NEW HAMBURG
MAXIMUM INSTANTANEOUS FLOW (CUBIC FEET PER SECOND)
1951 - 1979

YEAR	DATA	ORDERED	RANK	PROB.	RET. PERIOD
1951	5570	16000	1	.033	30.000
1952	3630	13700	2	.067	15.000
1953	3630	12800	3	.100	10.000
1954	6270	11800	4	.133	7.500
1955	4530	11400	5	.167	6.000
1956	7650	11000	6	.200	5.000
1957	4060	9880	7	.233	4.286
1958	3170	9540	8	.267	3.750
1959	5990	9220	9	.300	3.333
1960	8820	8820	10	.333	3.000
1961	2470	7650	11	.367	2.727
1962	7010	7030	12	.400	2.500
1963	9220	7010	13	.433	2.308
1964	3750	6950	14	.467	2.143
1965	9540	6800	15	.500	2.000
1966	2230	6290	16	.533	1.875
1967	11800	6270	17	.567	1.765
1968	9880	5990	18	.600	1.667
1969	6950	5570	19	.633	1.579
1970	4440	4530	20	.667	1.500
1971	3680	4440	21	.700	1.429
1972	6290	4060	22	.733	1.364
1973	6800	3750	23	.767	1.304
1974	11000	3680	24	.800	1.250
1975	16000	3630	25	.833	1.200
1976	11400	3630	26	.867	1.154
1977	12800	3170	27	.900	1.111
1978	7030	2470	28	.933	1.071
1979	13700	2230	29	.967	1.034

Table 3-12. Flood Estimates

```
              NITH RIVER AT NEW HAMBURG
   MAXIMUM INSTANTANEOUS FLOW (CUBIC FEET PER SECOND)
                     1951 - 1979

        SAMPLE STATISTICS

    MEAN =      7218          S.D. =    3623.6         C.S. =     .6730         C.K. =    3.0781

        SAMPLE STATISTICS (LOGS)

    MEAN =    8.7567          S.D. =      .5269        C.S. =    ¯.1903         C.K. =    2.5461

    SAMPLE MIN =    2230     SAMPLE MAX =   16000     N =  29

    PARAMETERS FOR GUMBEL I     A = .000357     U =      5559

    PARAMETERS FOR LOGNORMAL     M =  8.7567     S =  .5269

    PARAMETERS FOR THREE PARAMETER LOGNORMAL     A =      ¯170     M =  8.7868     S =  .5112

        STATISTICS OF LOG(X-A)

    MEAN =    8.7868          S.D. =      .5112        C.S. =    ¯.1639         C.K. =    2.5246

    PARAMETERS FOR LOG PEARSON III BY MOMENTS     A= ¯.0501 B=  110.4524 LOG(M)=14.2942     M =     1613982

    PARAMETERS FOR LOG PEARSON III BY MAXIMUM LIKELIHOOD     A = ¯.1413     B =  13.8620   LOG(M) =10.7155    M =        45050

    DISTRIBUTION STATISTICS     MEAN =  8.7567     S.D =  .5261     C.S. = ¯.5372



               GUMBEL I           LOGNORMAL       THREE PARAMETER                LOG PEARSON III
                                                     LOGNORMAL        MAXIMUM LIKELIHOOD         MOMENTS
  RETURN    FLOOD   ST.ERROR  FLOOD   ST. ERROR  FLOOD   ST. ERROR  FLOOD   ST. ERROR  FLOOD   ST. ERROR
  PERIOD   ESTIMATE  PERCENT ESTIMATE  PERCENT  ESTIMATE  PERCENT  ESTIMATE  PERCENT  ESTIMATE  PERCENT

    1.005     884             2030             1990              1720             1710
    1.050    2440             2670             2650              2490             2600
    1.250    4230             4080             4020              4160             4100
    2.000    6590             6350             6380              6660             6460
    5.000    9760     9.61    9900    11.40    9900    11.20     9970     9.69    9940    16.60
   10.000   11900    10.10   12500    13.20   12400    13.40    12000    10.20   12300    20.10
   20.000   13900    10.60   15100    15.00   15000    16.00    13800    11.40   14700    25.20
   50.000   16500    11.10   18700    17.20   18500    19.60    16000    14.00   17700    33.40
  100.000   18400    11.40   21600    18.80   21300    22.50    17500    16.30   20100    40.40
  200.000   20400    11.70   24700    20.30   24300    25.20    19000    19.00   22500    47.70
  500.000   23000    11.90   29000    22.20   28400    28.80    20800    22.60   25700    57.90
```

Table 3-13. Data Base and Plotting Positions

NITH RIVER AT CANNING
MAXIMUM INSTANTANEOUS FLOW (CUBIC FEET PER SECOND)
1948 - 1979

YEAR	DATA	ORDERED	RANK	PROB.	RET. PERIOD
~~1948~~	~~14900~~	~~14900~~	~~1~~	~~.030~~	~~33.000~~
1949	9910	14800	2	.061	16.500
1950	12600	13500	3	.091	11.000
~~1951~~	~~6720~~	~~12900~~	~~4~~	~~.121~~	~~8.250~~
1952	3970	12800	5	.152	6.600
1953	4590	12600	6	.182	5.500
~~1954~~	~~12900~~	~~12500~~	~~7~~	~~.212~~	~~4.714~~
1955	8940	11100	8	.242	4.125
1956	9420	10400	9	.273	3.667
~~1957~~	~~4850~~	~~10300~~	~~10~~	~~.303~~	~~3.300~~
1958	2360	10200	11	.333	3.000
1959	6510	9910	12	.364	2.750
~~1960~~	~~10200~~	~~9710~~	~~13~~	~~.394~~	~~2.538~~
1961	6500	9420	14	.424	2.357
1962	7320	8940	15	.455	2.200
~~1963~~	~~10300~~	~~7900~~	~~16~~	~~.485~~	~~2.063~~
1964	3610	7350	17	.515	1.941
1965	12500	7320	18	.545	1.833
~~1966~~	~~3540~~	~~7160~~	~~19~~	~~.576~~	~~1.737~~
1967	10400	7040	20	.606	1.650
1968	7040	7010	21	.636	1.571
~~1969~~	~~7010~~	~~6720~~	~~22~~	~~.667~~	~~1.500~~
1970	5100	6510	23	.697	1.435
1971	4280	6500	24	.727	1.375
~~1972~~	~~7350~~	~~5100~~	~~25~~	~~.758~~	~~1.320~~
1973	7160	4850	26	.788	1.269
1974	9710	4590	27	.818	1.222
~~1975~~	~~14800~~	~~4280~~	~~28~~	~~.848~~	~~1.179~~
1976	11100	3970	29	.879	1.138
1977	12800	3610	30	.909	1.100
~~1978~~	~~7900~~	~~3540~~	~~31~~	~~.939~~	~~1.065~~
1979	13500	2360	32	.970	1.031

Table 3-14. Flood Estimates

```
                NITH RIVER AT CANNING
MAXIMUM INSTANTANEOUS FLOW (CUBIC FEET PER SECOND)
                     1948 - 1979

     SAMPLE STATISTICS

MEAN =      8431         S.D. =    3511.3          C.S. =     .1865          C.K. =    2.3725

     SAMPLE STATISTICS (LOGS)

MEAN =    8.9424         S.D. =     .4704          C.S. =    ¯.6045          C.K. =    3.0630

SAMPLE MIN =    2360    SAMPLE MAX =   14900     N =  32

PARAMETERS FOR GUMBEL I     A =  .000328     U =      6740

PARAMETERS FOR LOGNORMAL      M =  8.9424     S =  .4704

PARAMETERS FOR THREE PARAMETER LOGNORMAL     A =   ¯14304     M = 10.0200     S =  .1552

     STATISTICS OF LOG(X-A)

MEAN =   10.0200         S.D. =     .1552          C.S. =    ¯.0529          C.K. =    2.3659

PARAMETERS FOR LOG PEARSON III BY MOMENTS     A= ¯.1422 B=   10.9470 LOG(M)=10.4989     M =        36276

PARAMETERS FOR LOG PEARSON III BY MAXIMUM LIKELIHOOD     A = ¯.4195     B =    1.1744   LOG(M) = 9.4351    M =        12520

DISTRIBUTION STATISTICS      MEAN =  8.9424     S.D =  .4546     C.S. =¯1.8455



              GUMBEL I            LOGNORMAL        THREE PARAMETER               LOG PEARSON III
                                                      LOGNORMAL       MAXIMUM LIKELIHOOD          MOMENTS
  RETURN    FLOOD    ST.ERROR  FLOOD    ST. ERROR  FLOOD    ST. ERROR  FLOOD    ST. ERROR  FLOOD    ST. ERROR
  PERIOD   ESTIMATE  PERCENT  ESTIMATE  PERCENT   ESTIMATE  PERCENT   ESTIMATE  PERCENT   ESTIMATE  PERCENT

   1.005     1660               2760               1750                   0                  2330
   1.050     3350               3530               3110                   0                  3280
   1.250     5290               5150               5420                   0                  5260
   2.000     7860               7650               8170                   0                  8020
   5.000    11300      8.50    11400      9.60    11300      7.25         0       .00    11400      16.40
  10.000    13600      9.15    14000     11.20    13100      7.92         0       .00    13400      21.20
  20.000    15800      9.64    16600     12.80    14700      9.12         0       .00    15200      28.10
  50.000    18600     10.20    20100     14.70    16600     11.00         0       .00    17200      39.10
 100.000    20700     10.50    22800     16.00    17900     12.60         0       .00    18500      48.50
 200.000    22900     10.80    25700     17.30    19200     14.10         0       .00    19800      58.50
 500.000    25700     11.10    29700     18.90    20800     16.20         0       .00    21300      72.40
```

coefficients of skew and kurtosis of the logarithmic transform with the addition lower boundary parameter should also approach 0.0 and 3.0.

The coefficients of skew and kurtosis for each station compare to the theoretical values as shown in Table 3.15.

It can be seen form the comparison of the coefficients of skew and kurtosis that of the Gumbel I, Log-Normal and Three-Parameter Log-Normal distributions, the Three-Parameter Log-Normal provides the best fit.

In all six cases, the fitting of the Log-Pearson Type III distribution produced an upper bound. The upper bound for both the moment and maximum likelihood fit at each station are shown below:

Table 3-16 - Upper Bound of Log Pearson Type III Distribution

Station	Upper Bound by Max.Likelihood	Ratio to Sample Max.	Upper Bound Est.By moments	Ratio To Sample Max.
Shand Dam	212632	8.2	300311	12.
Conestogo Dam	48176	2.3	273300	13.
Galt	91998	1.6	177825	3.0
Brantford	72009	1.1	193933	3.0
Guelph	12803	1.9	17085	2.5
New Hamburg	45050	2.8	1613982	100.
Canning	12520	0.8	36276	2.4

In the case of the maximum likelihood method of fit, the upper bound at five of the six stations sets an unreasonable restriction on the ordinate and produces much lower flows for low frequency events than do the other distributions. For Canning, the upper bound is lower than the sample maximum and the distribution cannot be used.

In the case of the moment method of fit, the upper bounds are generally much higher and have only marginal physical significance. The resulting flood estimates are similar to those produced by the Three Parameter Log-Normal distribution. It is felt that the Log-Pearson Type III with parameters estimated by moments provides a reasonable fit.

A comparison of the fit of the Three Parameter Log-Normal and the Log-Pearson Type III distribution moment solution is shown for each station in Appendix D.

A conclusion of a recent study done by Maclaren Engineering Planners & Scientists Inc., for The Ontario Ministry of Natural Resources, October, 1981 (5) states:

TABLE 3-15 Comparison of Coefficients of Skew and Kurtosis

Station	Coefficient of:	GUMBEL I		LOG-NORMAL		3-PARAMETER LOG-NORMAL	
		Sample	Theoretical	Sample	Theoretical	Sample	Theoretical
Shand Dam	skew	1.3024	1.14	-.2784	0.0	-.0579	0.0
	kurtosis	6.8325	5.4	3.1229	3.0	3.1979	3.0
Conestogo Dam	skew	.6534	1.14	-.2831	0.0	-.1082	0.0
	kurtosis	3.2168	5.4	2.7446	3.0	2.6381	3.0
Galt	skew	.5836	1.14	-.5431	0.0	-.0681	0.0
	kurtosis	3.2720	5.4	3.1476	3.0	2.7912	3.0
Brantford	skew	.3565	1.14	-.5525	0.0	-.0764	0.0
	kurtosis	2.5543	5.4	3.0631	3.0	2.5128	3.0
Guelph	skew	.7812	1.14	-.5602	0.0	-.0505	0.0
	kurtosis	4.1762	5.4	3.4246	3.0	3.0753	3.0
New Hamburg	skew	.6730	1.14	-.1903	0.0	-.1639	0.0
	kurtosis	3.0781	5.4	2.5461	3.0	2.5246	3.0
Canning	skew	.1865	1.14	-.6045	0.0	-.0529	0.0
	kurtosis	2.3725	5.4	3.0630	3.0	2.3659	3.0

"Estimates of peak flow using the Log-Pearson type III Distribution are somewhat more sensitive to variations in log skew than are estimates from the Three Parameter Log-Normal Distributions for similar variations in real skew. Generally speaking, the Three Parameter Log-Normal Distribution was found to be marginally superior compared to the Log-Pearson Type III Distribution".

This report makes the following recommendation:

"The Three Parameter Log-Normal Distribution is suggested for flood frequency analyses in the Province. If the coefficient of skew of the untransformed series is negative or the Three parameter Log-Normal distribution cannot otherwise be fitted, then the Log-Pearson Type III distribution should be used".

In view of the comparison of the distributions and the recommendation regarding the distribution to be used for future studies in Ontairo, the Three-Parameter Log Normal distribution was selected for use in this study.

3.3 Period of Analysis

In general, the period of record from 1948-1979 was used for the flood frequency analysis. (The period of record for Guelph and New Hamburg started with 1950 and 1951 respectively as the gauges were installed in those years). A common period of analysis at all stations was selected principally to aid in the development of flood frequency relationships for flood prone areas along the river (See 1.0 Background).

For the Grand River at Galt, only the last 32 years of 66 years of record were included in the analysis. A comparison of frequency curves developed from 32 years of record (1948-1979) and from 66 years of record (1914-1979) is shown in Appendix E. The shorter term analysis resulted in 12-14% higher flow estimates than the longer term analysis for the same return period. The frequency curve for the shorter term analysis lies within the upper 95% confidence interval of the frequency curve for the longer term analysis.

4.0 RESULTS

Flood frequency analysis was performed on maximum instantaneous spring freshet flows at the following long term gauge stations in the Grand River Basin:

Grand River at Shand Dam	1948-1979
Conestogo River at Conestogo Dam	1948-1979
Grand River at Galt	1948-1979
Grand River at Brantford	1948-1979
Speed River below Guelph	1950-1979
Nith River at New Hamburg	1951-1979
Nith River at Canning	1948-1979

The effects of regulation by the Shand, Conestogo, and Guelph Dams on the Grand, Conestogo, and Speed River stations were removed.

Where maximum instantaneous flows were not available, they were estimated based on the daily mean flow.

Analysis was performed using the Gumbel I, Log-Normal, Three Parameter Log-Normal, and Log-Pearson Type III distributions. The Three Parameter Log-Normal distribution was selected as providing the best fit.

Flood frequency curves for each station are presented in Figures 4-1 through 4-7. Upper and lower 95% confidence limits are shown on these curves.

The flow regime on the Grand, Conestogo, and Speed Rivers is greatly affected at the present time by the Shand, Conestogo, and Guelph Dams. When the frequency of flood flows of the existing river system is required, it is necessary to consider the effect of the existing reservoir system.

The existing reservoir operating procedure was modelled and applied in such a way that the effect of the reservoir system is considered in the consistent manner required for frequency analysis (see 1.0 Background). This was done using HEC-5, "Simulation of Flood Control and Conservation Systems"; a computer program prepared by the Hydrologic Engineering Centre, U.S. Army Corps of Engineers. As applied to this study, the model simulates the use of available flood storage in a system of reservoirs to minimize flood flows at selected downstream locations.

The initial water level assumed at the reservoirs was the April 1st target level set out in the existing operating policy. Storage available for flood control at this level is as follows:

Belwood - 10,000 acre-feet
Conestogo - 8,400 acre-feet
Guelph - 4,500 acre-feet

The effect of this flood control storage was determined for the floods having 5,10,20,50, and 100-year return periods with no reservoirs in place. The resulting flood flows were assumed to have the same return periods.

The methodology used and the assumptions made are discussed in more detail in Technical Report #38, "Reservoir Operations".

For completeness, the flood frequency curves for flows regulated by the existing reservoir system are presented in Figures 4-8 through 4-12 for the following stations:

Grand River at Shand Dam
Conestogo River at Conestogo Dam
Grand River at Galt
Grand River at Brantford
Speed River below Guelph

Figure 4-1. GRAND RIVER AT SHAND DAM
Flood Frequency Analysis
Maximum Instantaneous Flow 1948-1979

Natural Flow with 95% Confidence Limits

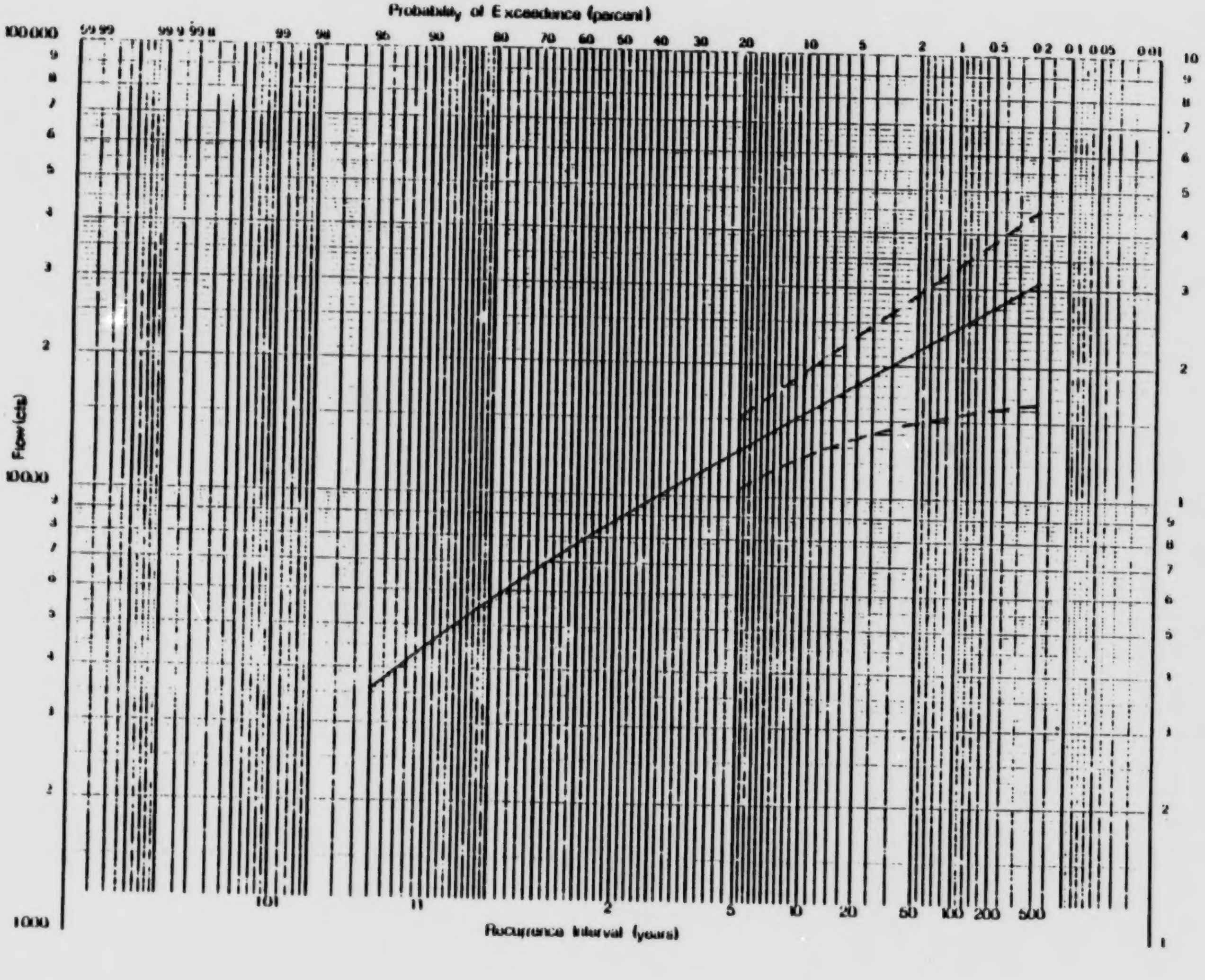


Figure 4-2. CONESTOGO RIVER AT CONESTOGO DAM
Flood Frequency Analysis
Maximum Instantaneous Flow 1948-1979

Natural Flow with 95% Confidence Limits

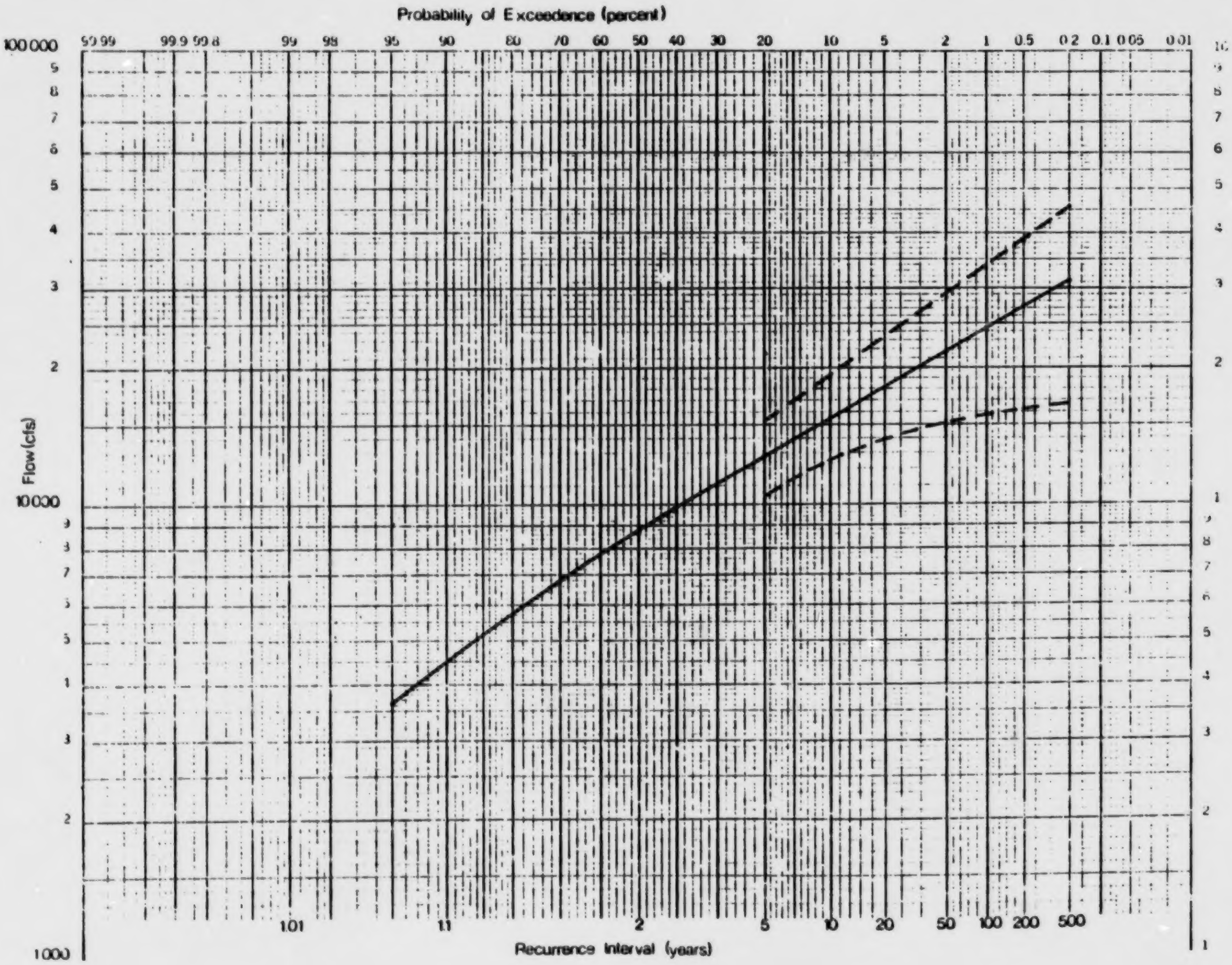


Figure 4-3. GRAND RIVER AT GALT
Flood Frequency Analysis
Maximum Instantaneous Flow 1948-1979

Natural Flow with 95% Confidence Limits

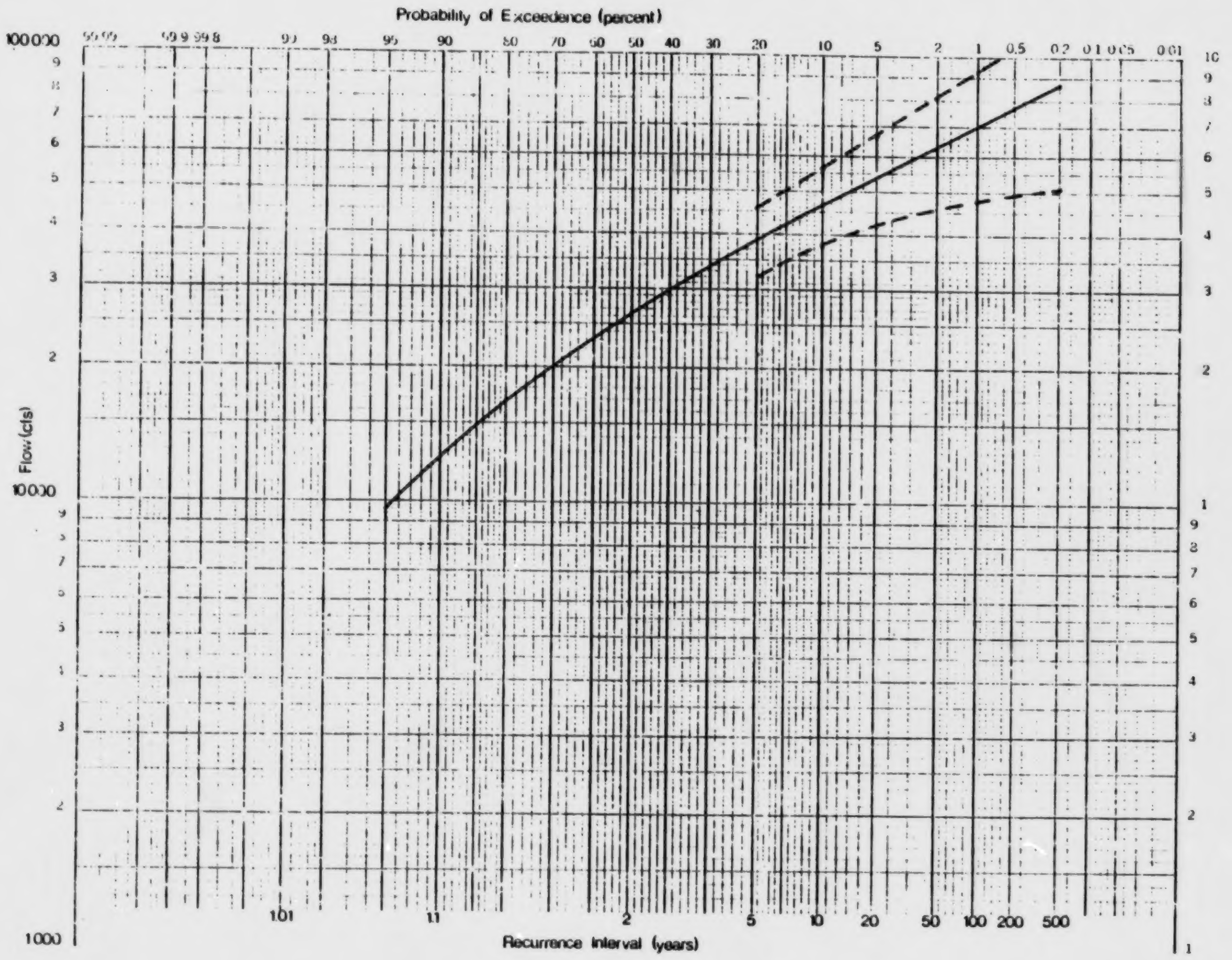


Figure 4-4. GRAND RIVER AT BRANTFORD
Flood Frequency Analysis
Maximum Instantaneous Flow 1948-1979

Natural Flow with 95% Confidence Limits

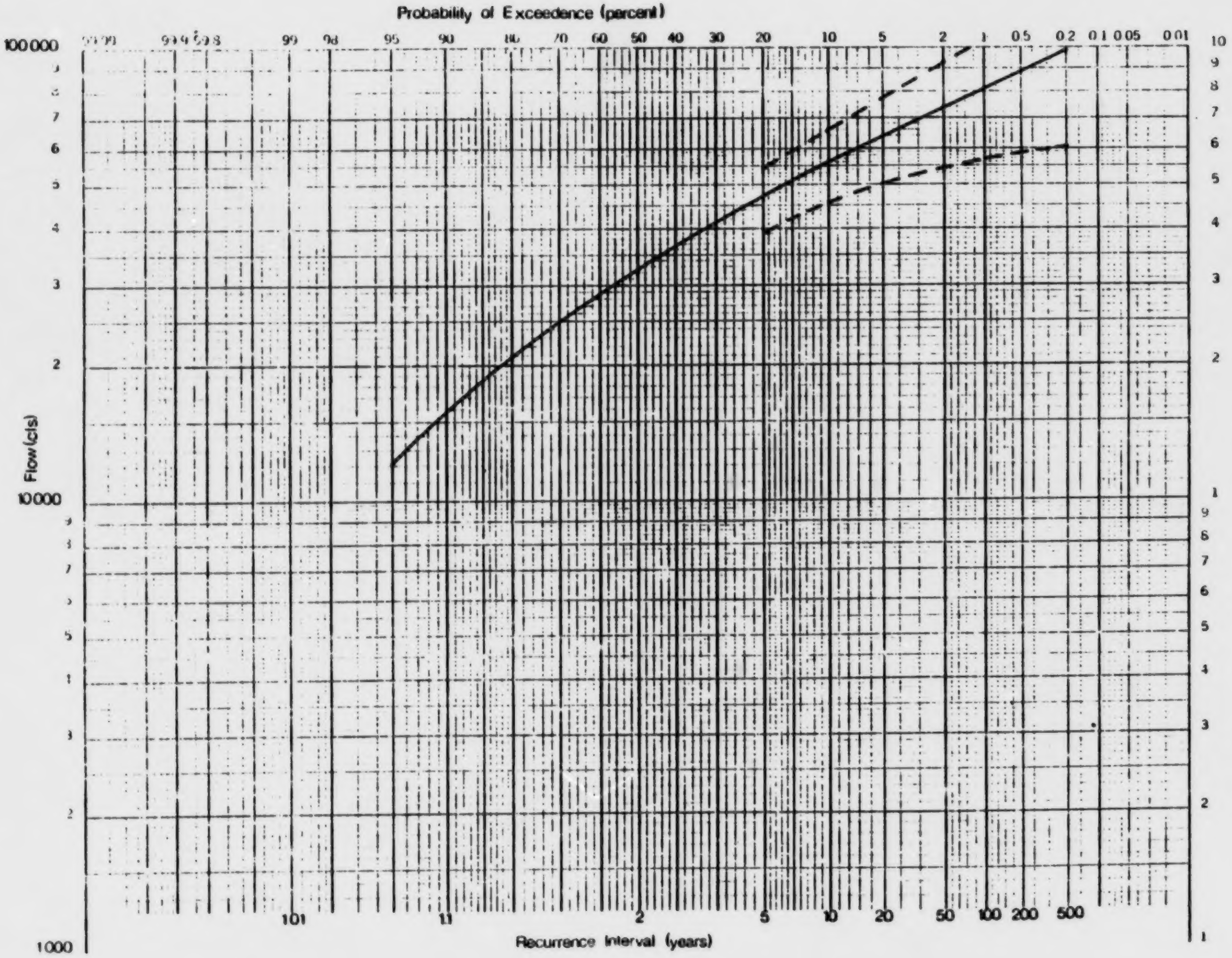


Figure 4-5. SPEED RIVER BELOW GUELPH
Flood Frequency Analysis
Maximum Instantaneous Flow 1950-1979

Natural Flow with 95% Confidence Limits

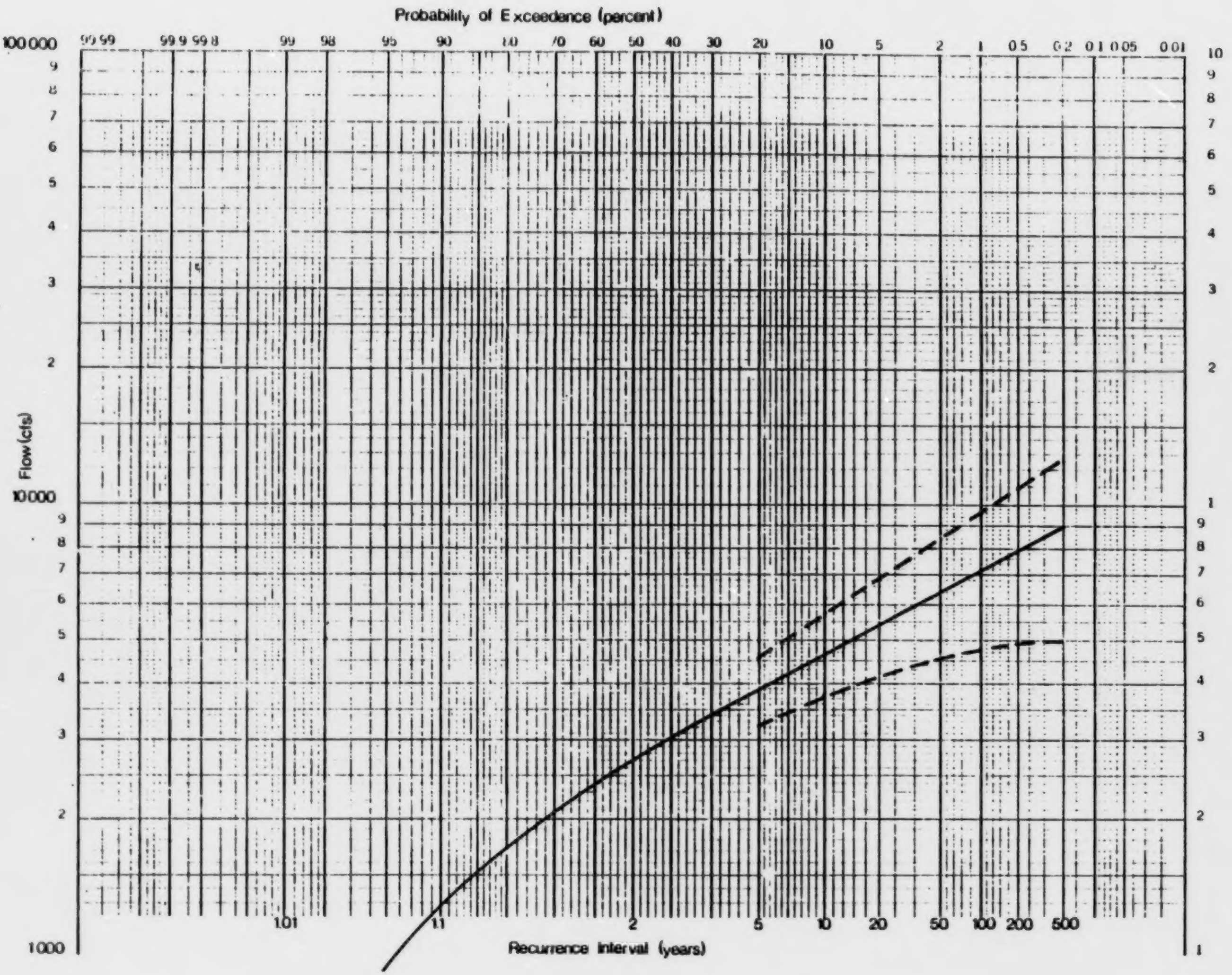


Figure 4-6. NITH RIVER AT NEW HAMBURG
Flood Frequency Analysis
Maximum Instantaneous Flow 1951-1979

Natural Flow with 95% Confidence Limits

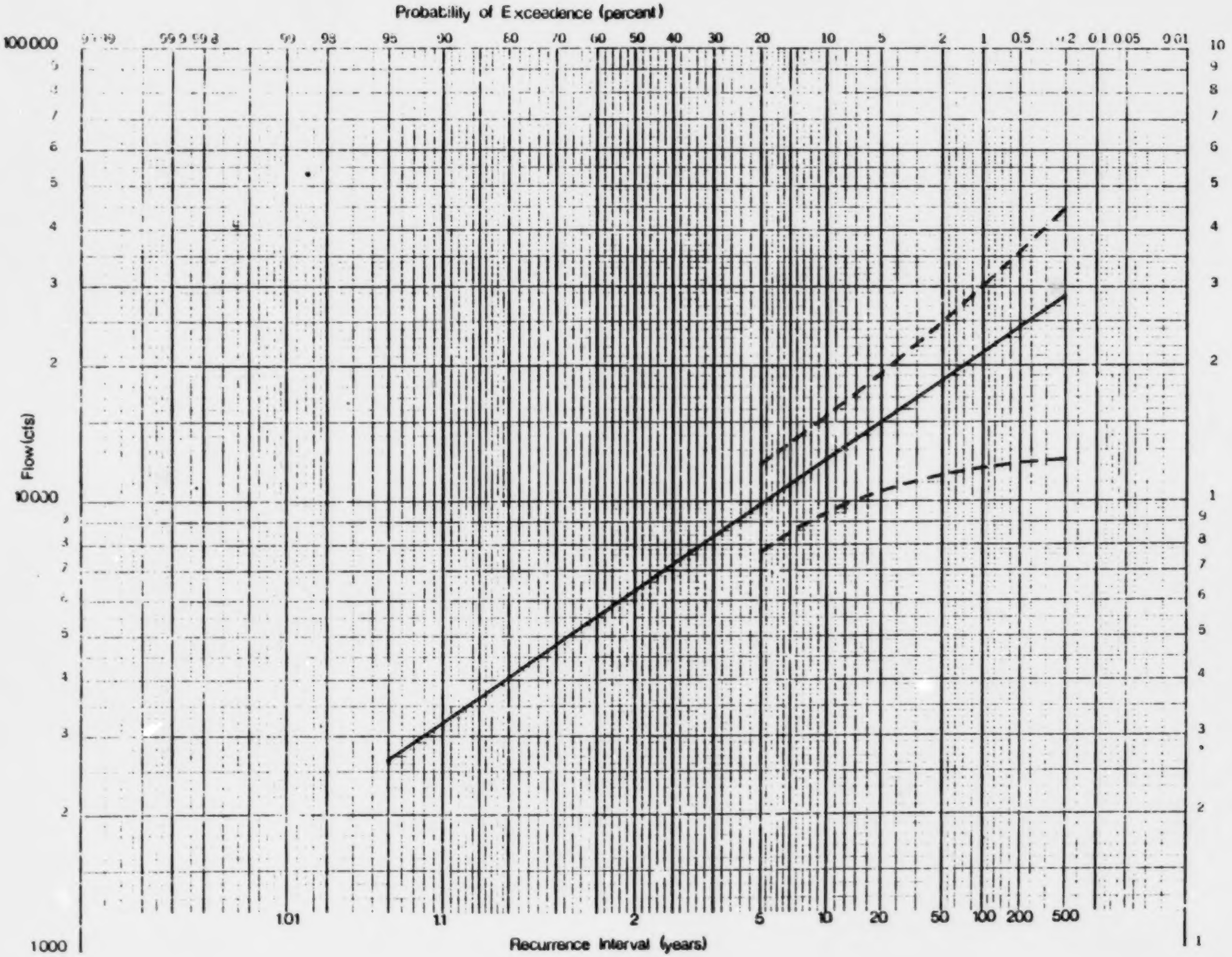


Figure 4-7. NITH RIVER AT CANNING
Flood Frequency Analysis
Maximum Instantaneous Flow 1948-1979

Natural Flow with 95% Confidence Limits

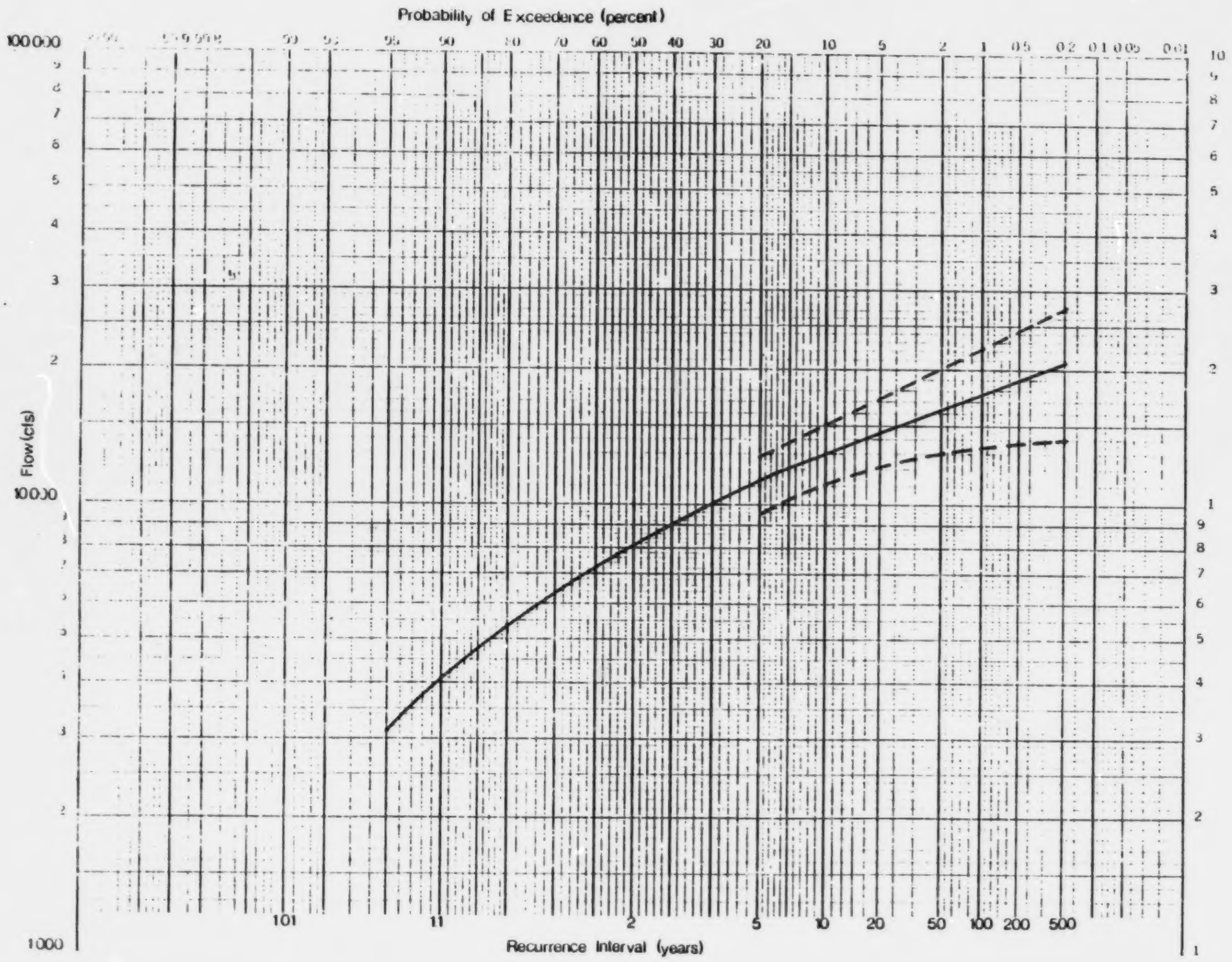


Figure 4-8. GRAND RIVER AT SHAND DAM
Flood Frequency Analysis
Maximum Instantaneous Flow 1948-1979

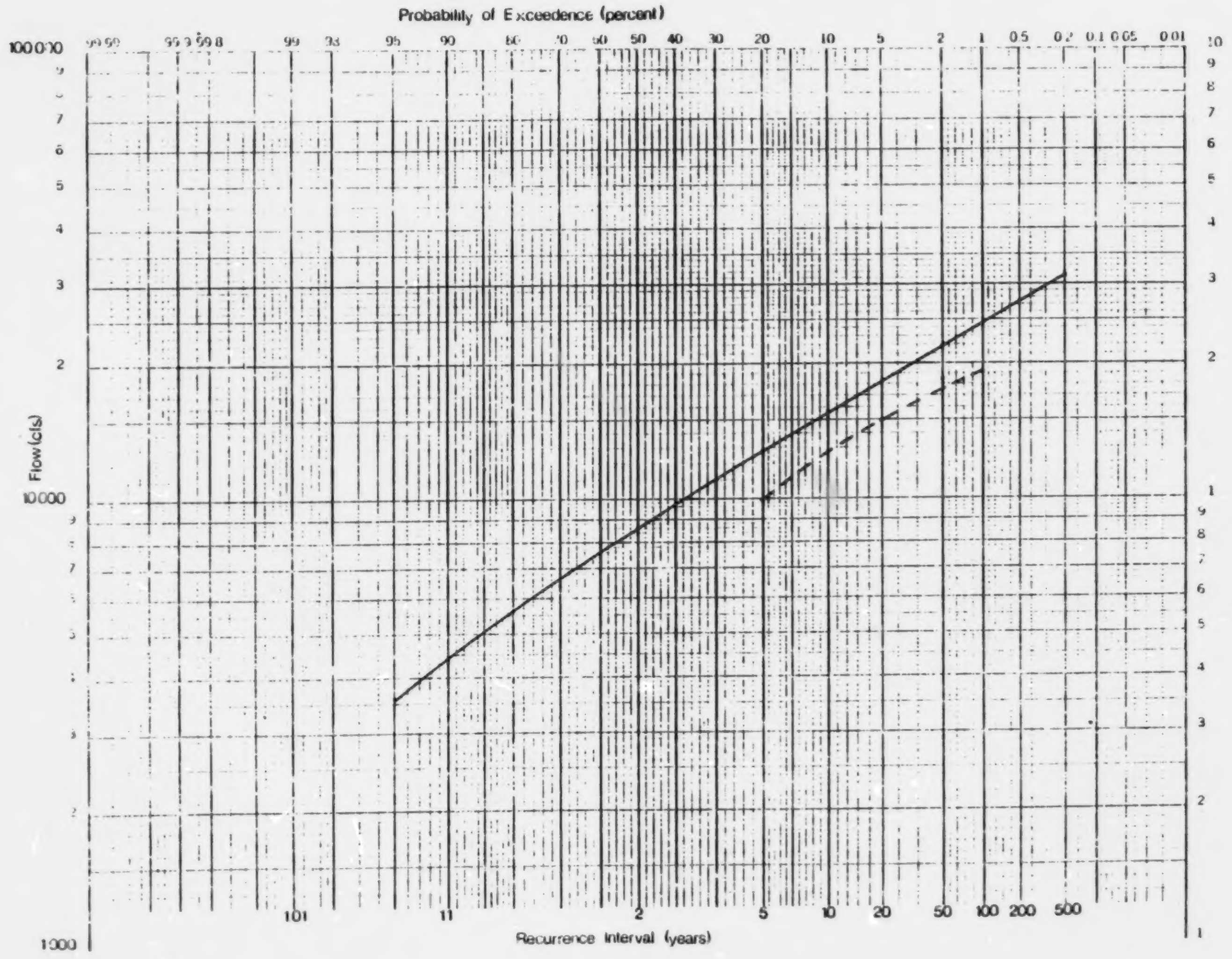


Figure 4-9. CONESTOGO RIVER AT CONESTOGO DAM
Flood Frequency Analysis
Maximum Instantaneous Flow 1948-1979

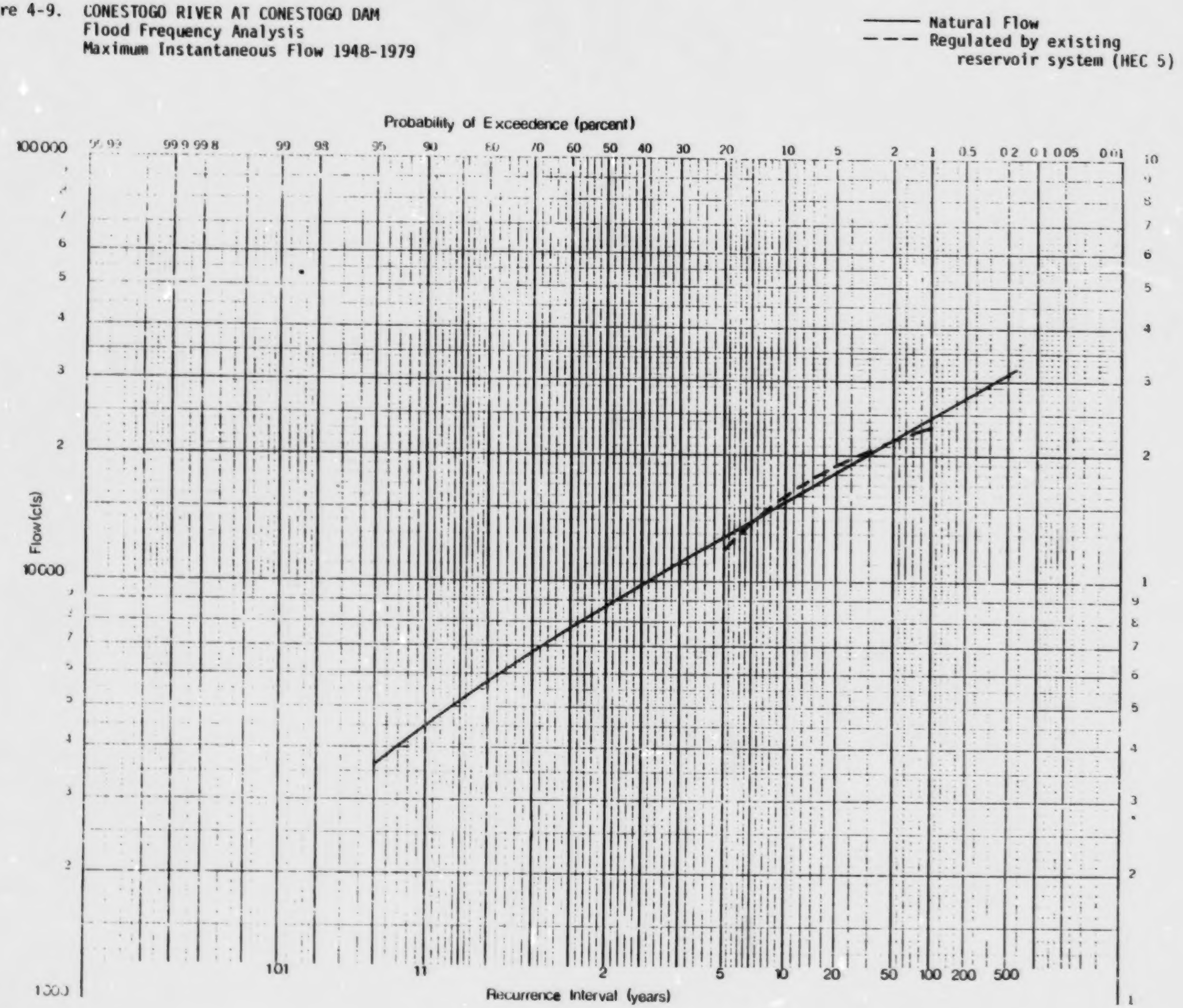


Figure 4-10. GRAND RIVER AT GALT
Flood Frequency Analysis
Maximum Instantaneous Flow 1948-1979

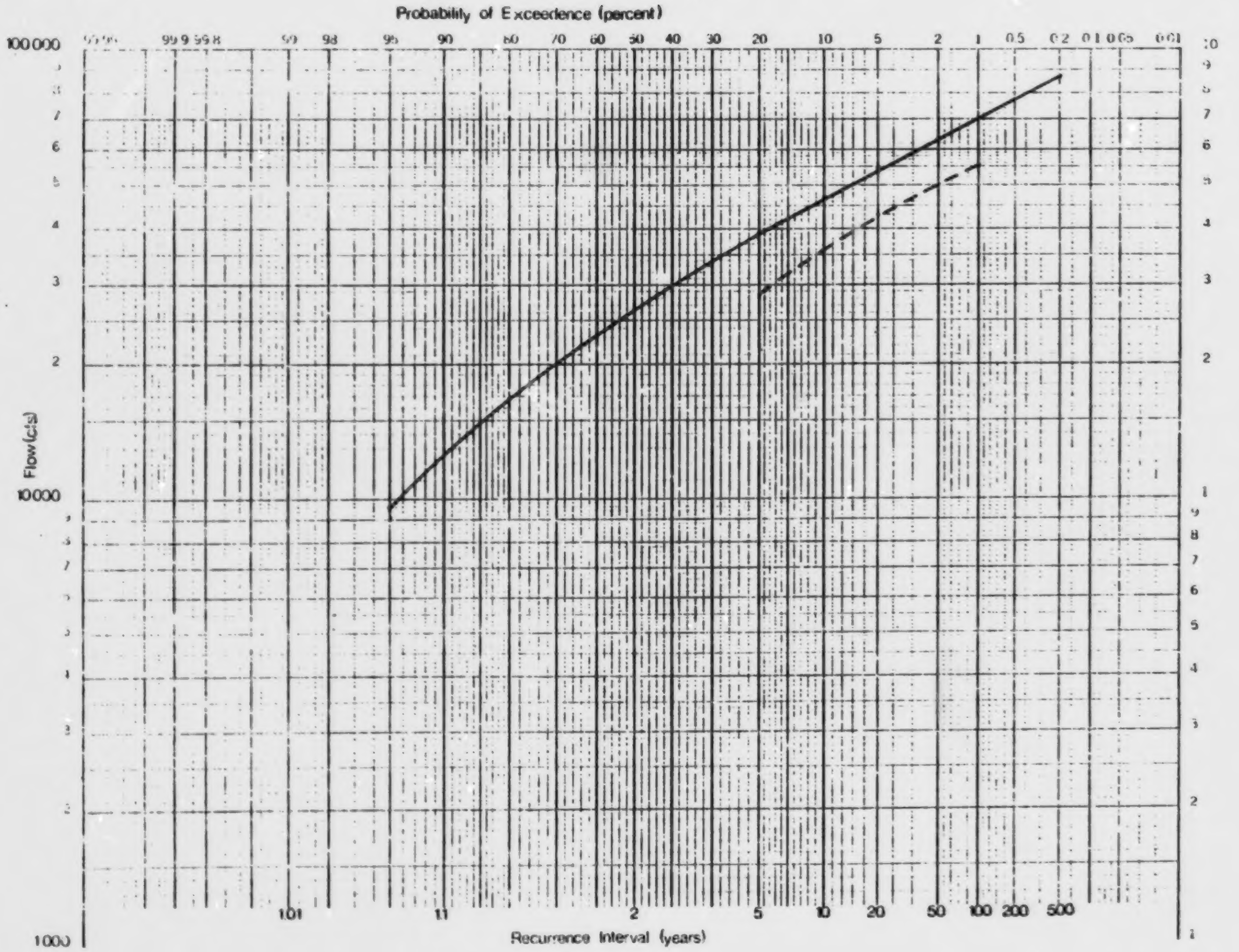


Figure 4-11. GRAND RIVER AT BRANTFORD
Flood Frequency Analysis
Maximum Instantaneous Flow 1948-1979

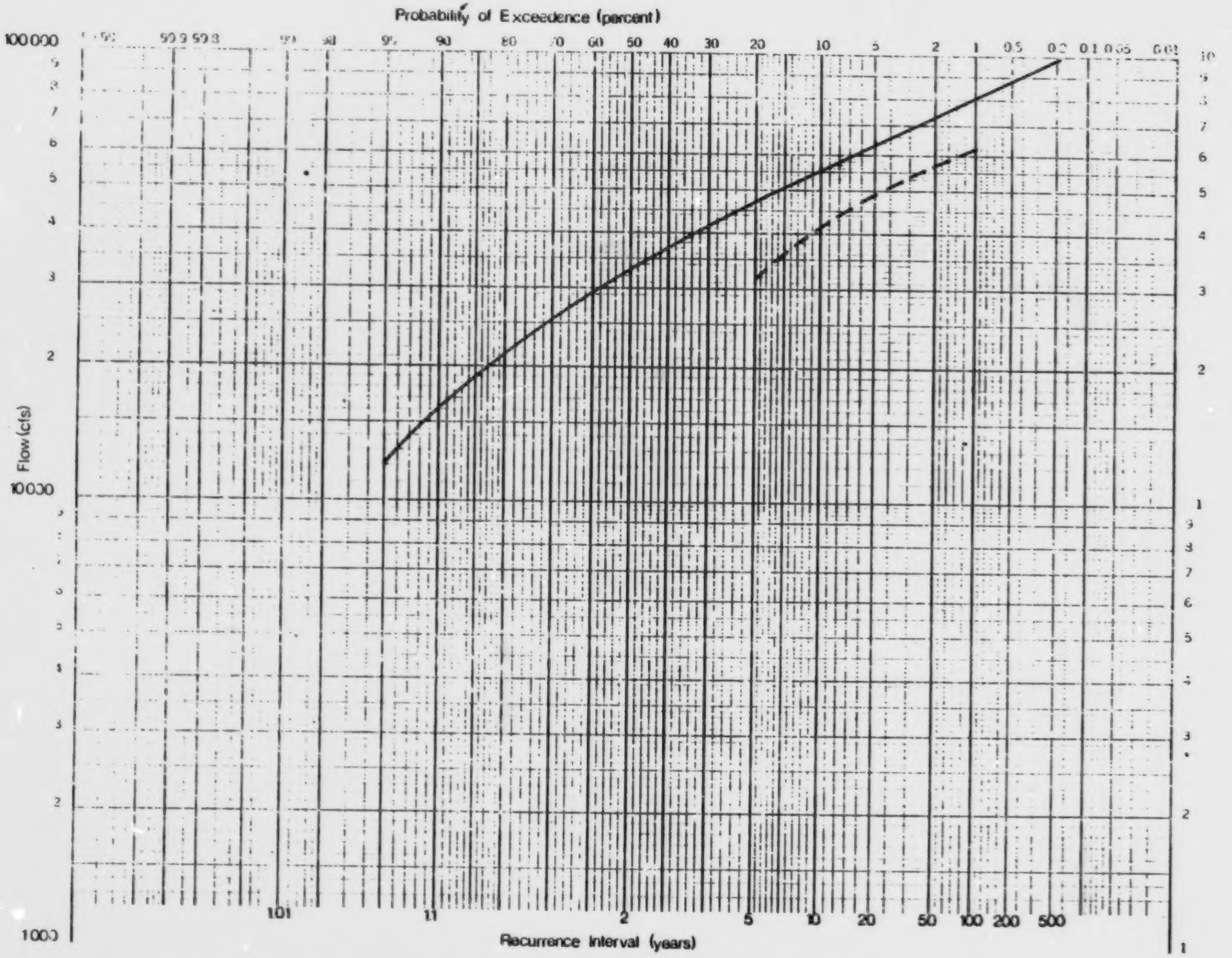
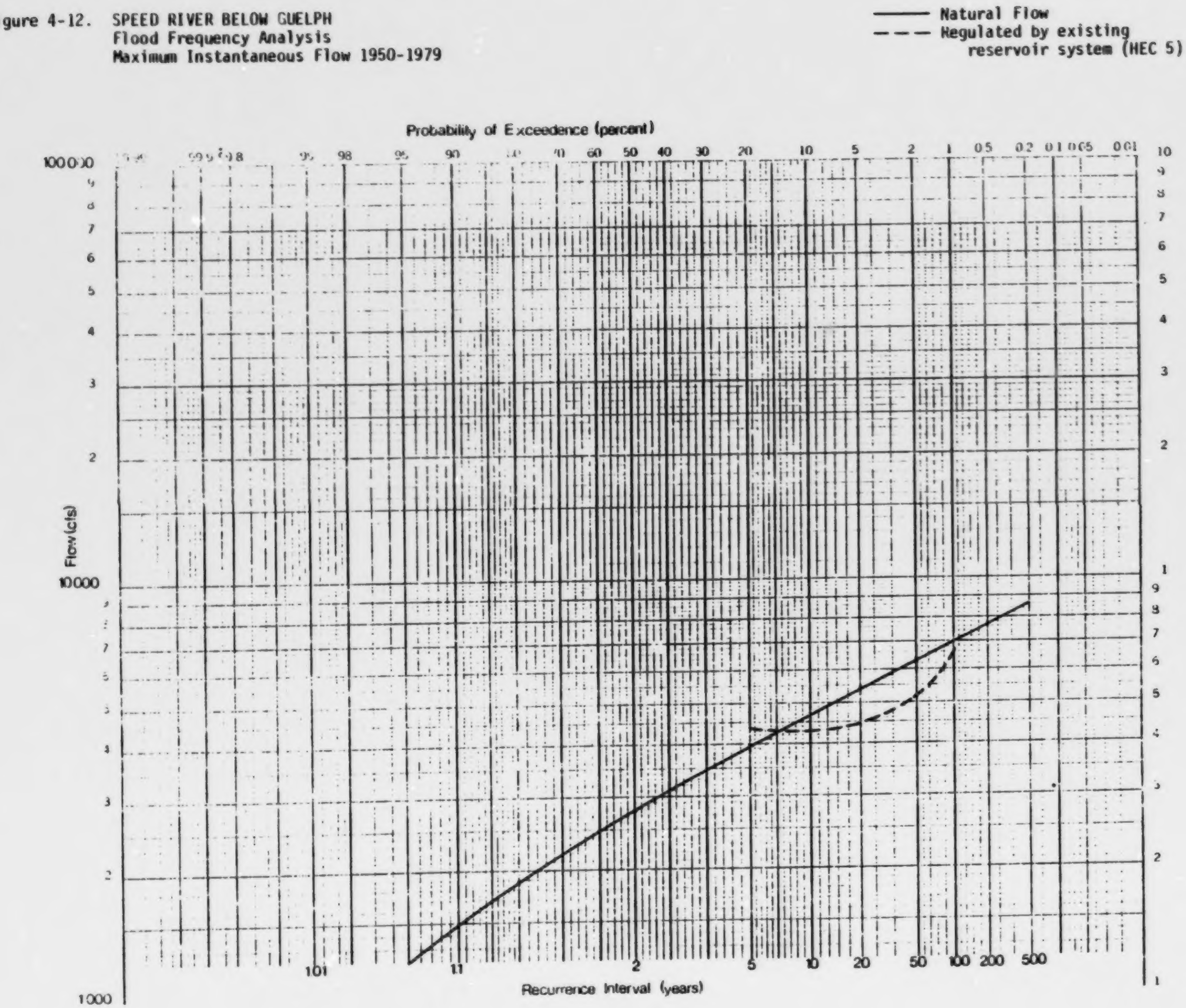


Figure 4-12. SPEED RIVER BELOW GUELPH
Flood Frequency Analysis
Maximum Instantaneous Flow 1950-1979



5.0 LIST OF REFERENCES

1. Automated Business and Engineering Ltd. "Choice of Statistical Distributions for Flood Flow Frequency Analysis in Ontario", Vol. I-III, Prepared for the Conservation Authorities and Water Management Branch, Ontario Ministry of Natural Resources, August 1980.

2. Bobee, B., "The Log Pearson Type 3 Distribution and its Application in Hydrology", Water Resources Research, 11(5), 681-689, 1975.

3. Condie, R., G.A. Nix, L.G. Boone, "Program FDRPFFA - Flood Damage Reduction Program Flood Frequency Analysis", Engineering Hydrology Section, Engineering and Development Division, Water Planning and Management Branch, Inland Waters Directorate, Environment Canada, January 1977.

4. Dickinson, W.T. and T.C. Sharma, "Detecting Hydrologic Changes - A Case Study", Canadian Hydrology Symposium Proceedings, May, 1980.

5. MacLaren Engineers Planners & Scientists Inc., "Statistical Hydrology - Regionalization of the Coefficient of Skew for the Province of Ontario", Prepared for the Conservation Authorities and Water Management Branch, Ontario Ministry of Natural Resources, October 1981.

6. Sangal, B.P., and Kallio, R.W., "Magnitude and Frequency of Floods in Southern Ontario" Technical Bulleting Series No. 99, Inland Waters Directorate, Water Planning and Management Branch, Ottawa, Canada 1977.

APPENDIX A

MAXIMUM INSTANTANEOUS FLOWS VERSUS DAILY MEAN FLOWS

TABLE A-1 MAXIMUM INSTANTANEOUS FLOWS VERSUS DAILY MEAN FLOWS

SHAND DAM INFLOW

N = 23
x = 6080
y = 9450
y = 276 + 1.510x

	MAX. DAILY	MAX. INST.
1942		
1943		
1944		
1945		
1946		
1947		
1948	16900	25800
1949	5030	7870
1950	8030	12400
1951	6580	10200
1952	5460	8520
1953	2950	4730
1954	6380	9900
1955	5710	11700
1956	8410	13300
1957	2790	4490
1958	1990	3280
1959	5760	8040
1960	8540	11200
1961	2440	3460
1962	5010	10400
1963	6680	9680
1964	3140	4630
1965	5100	5980
1966	2240	3220
1967	4590	8510
1968	3720	5580
1969	3320	5710
1970	5610	8300
1971	7270	8290
1972	9930	14100
1973	4550	7130
1974	8580	14800
1975	8410	15600
1976	7850	14100
1977	9010	13100
1978	6610	8120
1979	7310	12500

Maximum instantaneous flows enclosed in boxes are estimated based on corresponding maximum daily flows.

Figure A-1.

Shand Dam Inflow

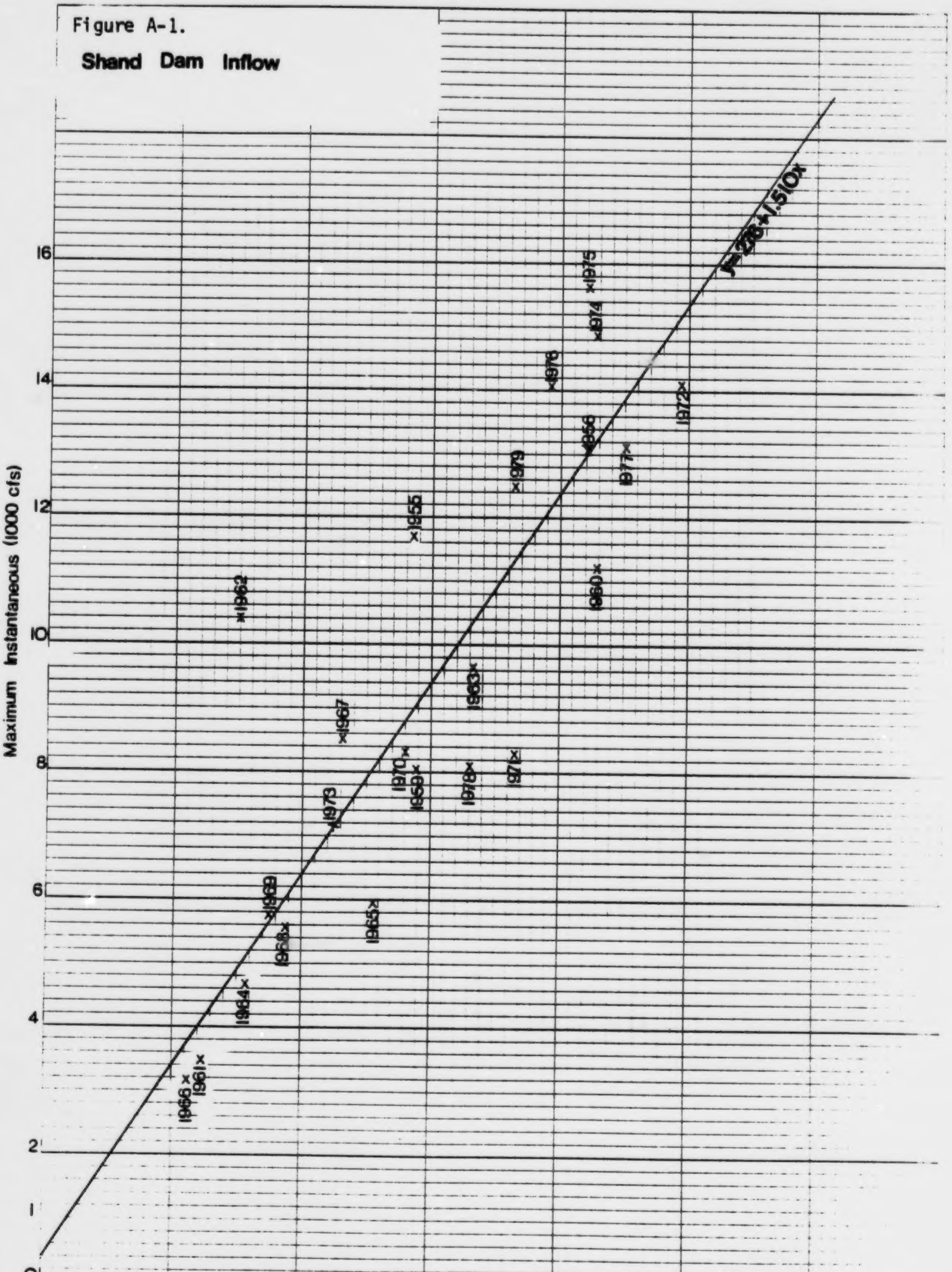


TABLE A-2 MAXIMUM INSTANTANEOUS FLOWS VERSUS DAILY MEAN FLOWS

CONESTOGO DAM INFLOW

	MAX. DAILY	MAX. INST.
1948	7960	12600
1949	4050	7900
1950	5320	9410
1951	3390	7110
1952	3080	6750
1953	1300	4630
1954	5720	9890
1955	1580	4960
1956	5520	9650
1957	1290	4620
1958	353	3500
1959	3490	6350
1960	9950	13000
1961	1910	3650
1962	6460	12800
1963	7840	11700
1964	3740	8340
1965	5080	15600
1966	1670	3130
1967	4790	12000
1968	3580	7440
1969	3410	8280
1970	4220	6720
1971	4540	5250
1972	5200	8590
1973	3830	8990
1974	4750	7670
1975	12300	20800
1976	11600	17200
1977	12930	15320
1978	6520	8460
1979	8510	13600

$N = 21$
$x = 6020$
$y = 10200$
$y = 3080 + 1.190x$

Maximum instantaneous flows enclosed in boxes are estimated based on corresponding maximum daily flows.

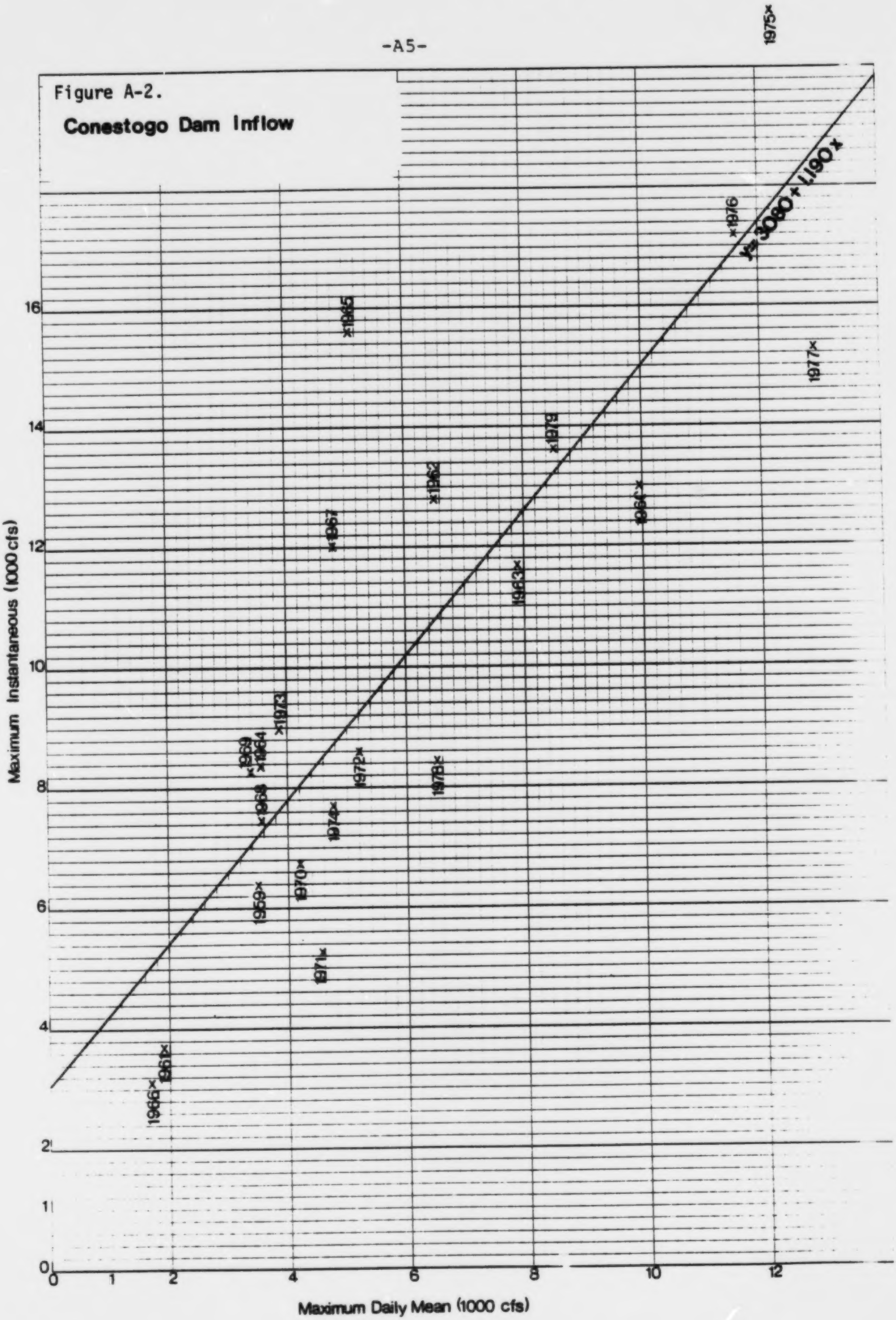


Figure A-2.

Conestogo Dam Inflow

TABLE A-3 MAXIMUM INSTANTANEOUS FLOWS VERSUS DAILY MEAN FLOWS

NATURAL FLOW-GRAND RIVER AT GALT

	MAX. DAILY	MAX. INST.
1943	28700	37300
1944	17600	22700
1945	25600	33200
1946	20200	26100
1947	37200	48500
1948	44100	57600
1949	23200	30100
1950	27600	35900
1951	23400	30300
1952	16600	21400
1953	10700	13600
1954	18500	23900
1955	10500	17200
1956	24900	31100
1957	9610	12200
1958	5850	7210
1959	27600	32900
1960	38600	46300
1961	8500	10400
1962	23100	29700
1963	25400	28200
1964	11500	15000
1965	20100	26700
1966	7100	9380
1967	28300	37000
1968	13100	14600
1969	17500	23800
1970	13600	17900
1971	20300	21400
1972	27800	32400
1973	22600	28200
1974	26500	33400
1975	31500	58700
1976	30800	42800
1977	32800	40400
1978	24100	26600
1979	23000	36000

N = 23
x = 22100
y = 28700
y = 1.317 x -499

Maximum instantaneous flows enclosed in boxes are estimated based on corresponding maximum daily flows.

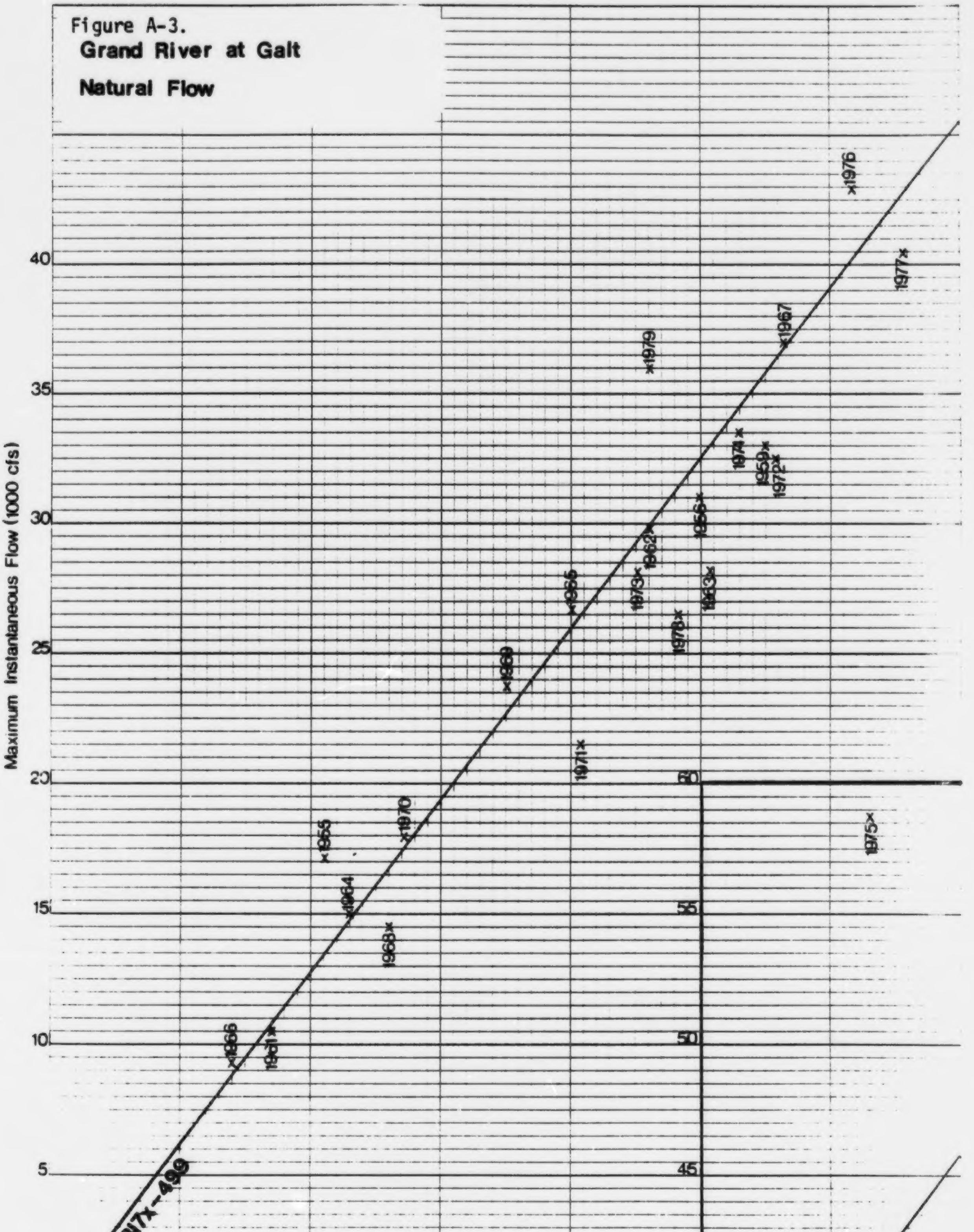


Figure A-3.
Grand River at Galt
Natural Flow

TABLE A-4 MAXIMUM INSTANTANEOUS FLOWS VERSUS DAILY MEAN FLOWS

NATURAL FLOW-GRAND RIVER AT BRANTFORD

	MAX DAILY	MAX. INST.
1948	52900	61100
1949	24600	29800
1950	45700	53100
1951	30400	36200
1952	20700	25500
1953	14400	18600
1954	32300	38300
1955	23300	26000
1956	38500	40500
1957	16000	20300
1958	7800	11300
1959	31000	35700
1960	45400	55500
1961	12500	14100
1962	24700	34600
1963	32600	36900
1964	13000	17000
1965	27700	33600
1966	7320	9320
1967	20700	36500
1968	16910	18400
1969	20000	24300
1970	12700	17500
1971	22400	24300
1972	33300	38000
1973	25400	33800
1974	28500	36000
1975	49600	64000
1976	44400	52200
1977	50000	53200
1978	30100	33100
1979	47100	52700

N = 23
x = 28600
y = 34200
y = 2670 + 1.104x

Maximum instantaneous flows enclosed in boxes are estimated based on corresponding maximum daily flows.

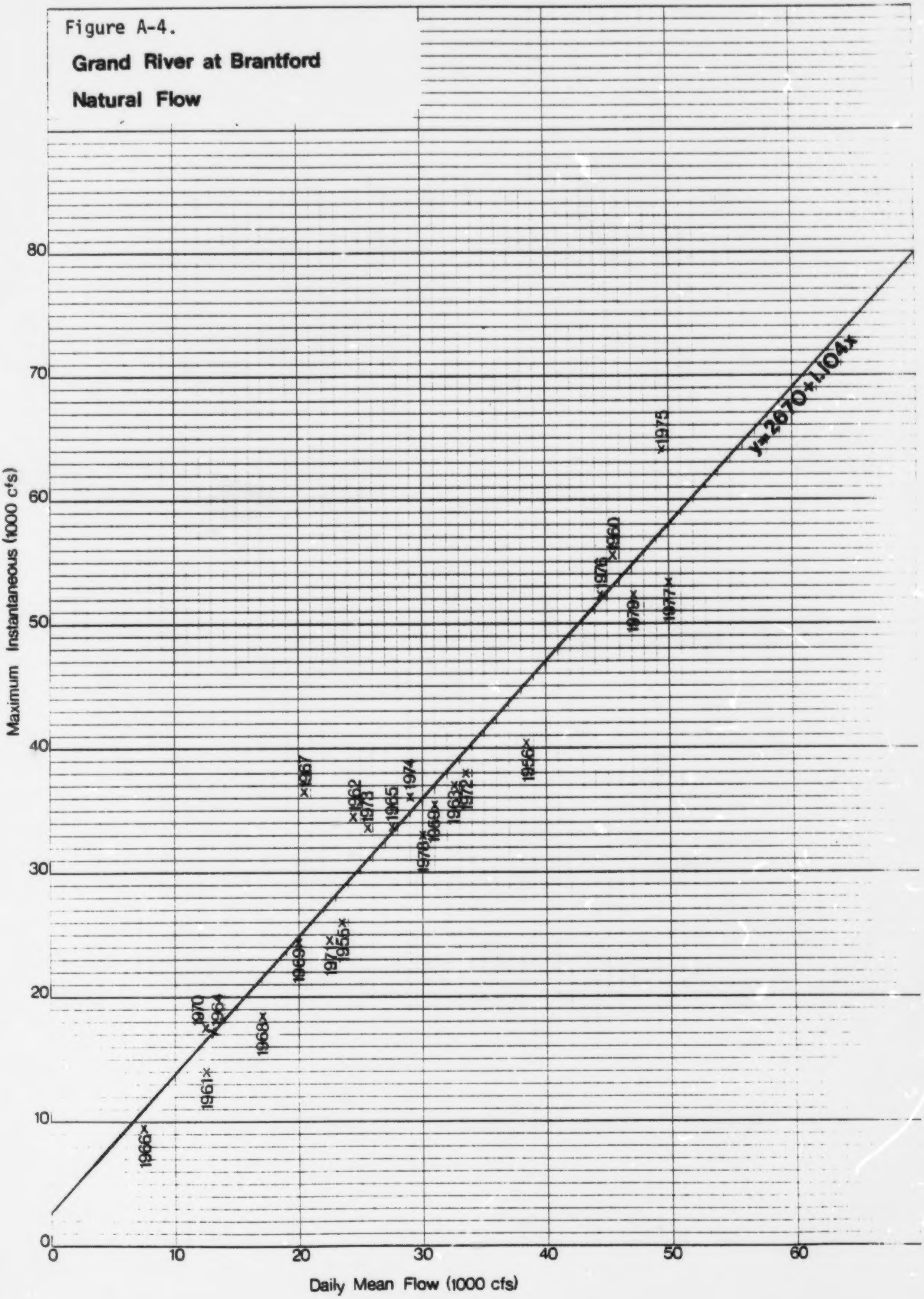


Figure A-4.

Grand River at Brantford

Natural Flow

TABLE A-5 MAXIMUM INSTANTANEOUS FLOWS VERSUS DAILY MEAN FLOWS

NATURAL FLOW-SPEED RIVER BELOW GUELPH

$N = 19$
$x = 2460$
$y = 2870$
$y = 91.5 + 1.128x$

	MAX. DAILY	MAX. INST.
1950	----	6720
1951	3000	3780
1952	1580	2250
1953	1390	2080
1954	2460	3010
1955	2060	2660
1956	3260	3670
1957	1210	1930
1958	550	1360
1959	2940	3420
1960	2860	3010
1961	1190	1910
1962	1980	2170
1963	2840	3000
1964	870	1020
1965	2760	3460
1966	820	920
1967	3780	4810
1968	1330	2300
1969	2520	2750
1970	1400	1640
1971	1530	1700
1972	3480	3730
1973	2670	2930
1974	3310	3430
1975	3370	4480
1976	----	3500E
1977	3400	3700
1978	2010	2280
1979	2800	3340

Maximum instantaneous flows enclosed in boxes are estimated based on corresponding maximum daily flows.

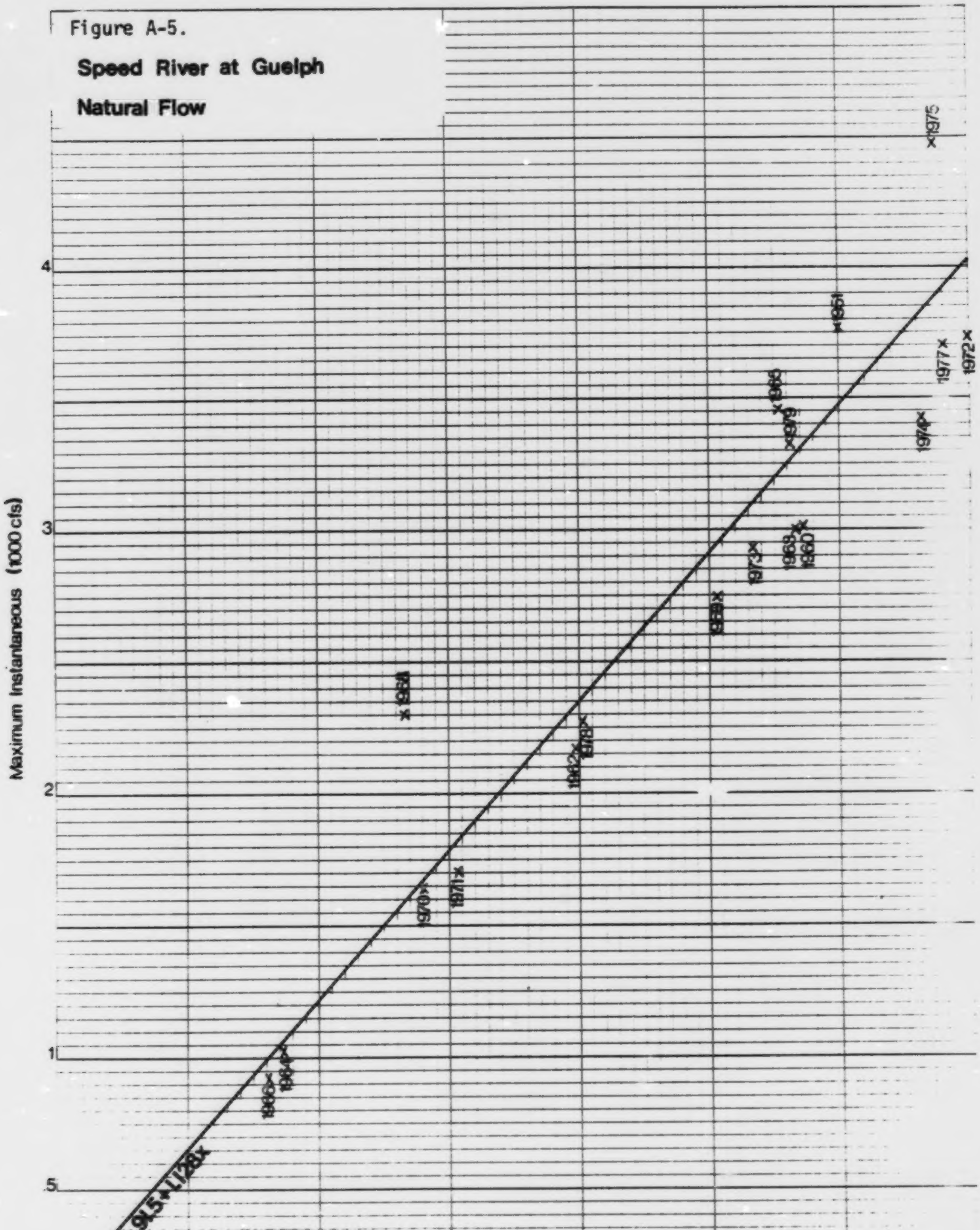


Figure A-5.

Speed River at Guelph

Natural Flow

TABLE A-6 MAXIUMUM INSTANTANEOUS FLOWS VERSUS DAILY MEAN FLOWS

NITH RIVER AT NEW HAMBURG

	MAX. DAILY	MAX. INST.
1951	3490	5570
1952	2750	3630
1953	2620	$\boxed{3630}$
1954	4470	$\boxed{6270}$
1955	3250	$\boxed{4530}$
1956	4990	7650
1957	2920	$\boxed{4060}$
1958	2300	$\boxed{3170}$
1959	5090	5990
1960	7130	8820
1961	2180	2470
1962	5480	7010
1963	7520	9220
1964	2700	$\boxed{3750}$
1965	5680	9450
1966	1640	$\boxed{2230}$
1967	7460	11800
1968	5140	9880
1969	5080	6950
1970	3780	4440
1971	3030	3680
1972	5090	6290
1973	4680	6800
1974	6920	11000
1975	10700	16000
1976	9380	11400
1977	9110	12800
1978	4910	7030
1979	9390	13700

N = 22
x = 5860
y = 8260
y = 1:429 x - 113

Maximum instantaneous flows enclosed in boxes are estimated based on corresponding maximum daily flows.

Figure A-6.

Nith River at New Hamburg

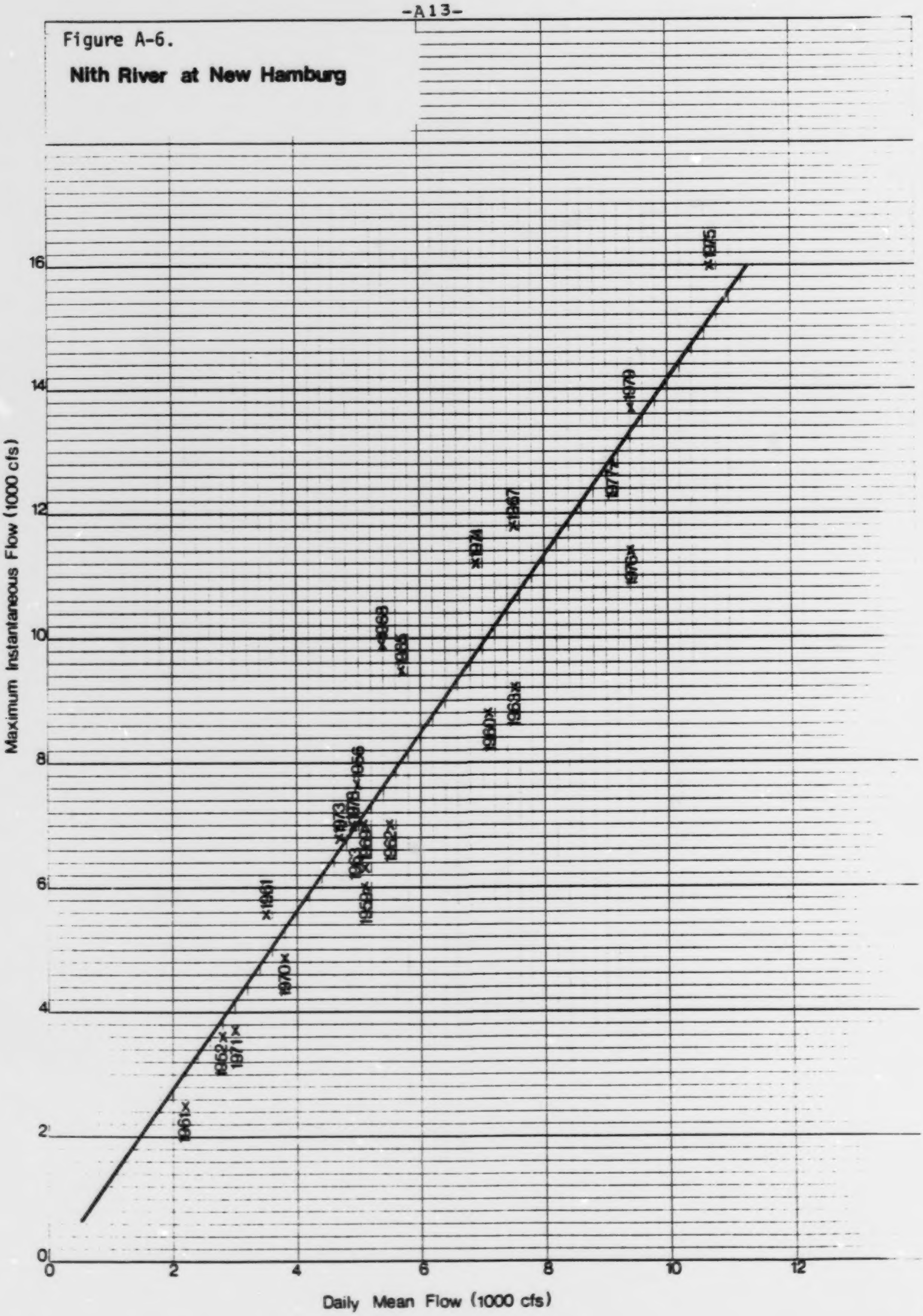


TABLE A-7 MAXIMUM INSTANTANEOUS FLOWS VERSUS DAILY MEAN FLOWS

NITH RIVER AT CANNING

	MAX. DAILY	MAX. INST.
1948	10500	14900
1949	6130	9910
1950	11600	12600
1951	5700	6720
1952	3900	3970
1953	2680	4590
1954	10600	12900
1955	7110	8940
1956	8100	9420
1957	4050	4850
1958	1980	2360
1959	6000	6510
1960	8760	10200
1961	2920	6500
1962	5910	7320
1963	8590	10300
1964	2560	$\boxed{3610}$
1965	8380	12500
1966	2500	$\boxed{3540}$
1967	7440	10400
1968	5500	$\boxed{7040}$
1969	5860	7010
1970	4550	5100
1971	4070	4280
1972	6600	7350
1973	5260	7160
1974	8460	9710
1975	11300	14800
1976	9770	11100
1977	10900	12800
1978	6180	7900
1979	10900	13500

$N = 29$
$x = 7040$
$y = 8830$
$y = 634 + 1.164x$

Maximum instantaneous flows enclosed in boxes are estimated based on corresponding maximum daily flows.

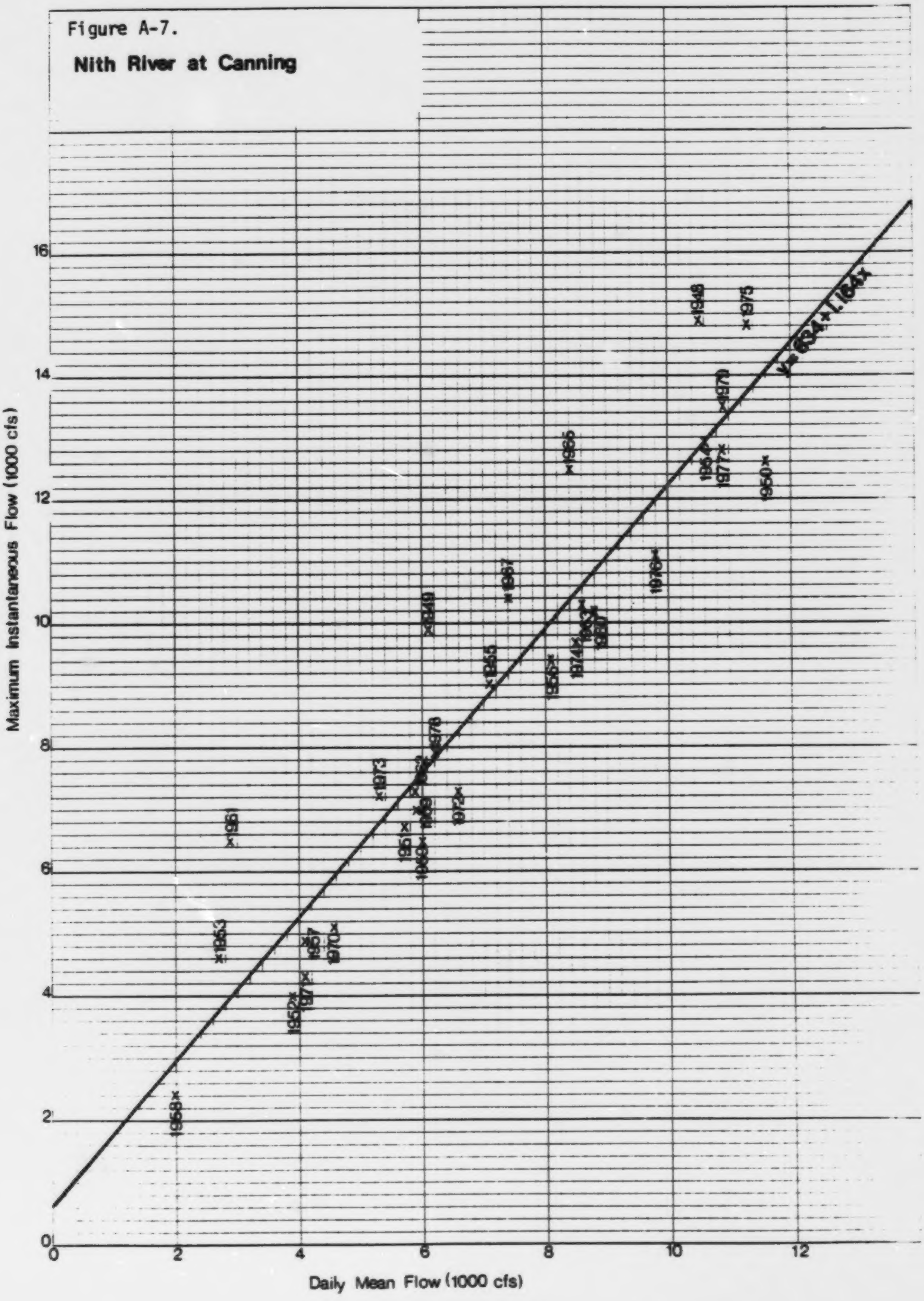


Figure A-7.

Nith River at Canning

APPENDIX B

MAXIMUM NATURAL FLOW VERSUS CORRESPONDING OBSERVED FLOW

TABLE B-1 MAXIMUM NATURAL FLOW VERSUS CORRESPONDING OBSERVED FLOW

GRAND RIVER AT GALT

YEAR	DATE	MAXIMUM NATURAL FLOW PEAK (cfs)	CORRESPONDING OBSERVED FLOW PEAK (cfs)
1943	Apr 2	**37300**	-----
1944	Mar 25	**22700**	28900
1945	Mar 16	**33200**	19200
1946	Mar 7	**26100**	30200
1947	Apr 12	**48500**	41400
1948	Mar 20	**57600**	46300
1949	Mar 23	**30100**	30400
1950	Apr 5	**35900**	38600
1951	Mar 31	**30300**	23800
1952	Apr 2	**21400**	17300
1953	May 3	**13600**	15000 *
1954	Apr 17	**23900**	20200 *
1955	Mar 12	17200	15400
1956	Apr 5	31100	30800
1957	Feb 27	**12200**	14900 *
1958	Mar 29	**7210**	4190
1959	Apr 6	32900	20100
1960	Apr 4	46300	28600
1961	Feb 25	10400	6680
1962	Mar 30	29700	13800
1963	Mar 28	28200	14400
1964	Apr 8	15000	5730
1965	Apr 8	26700	22100
1966	Mar 19	9380	4590
1967	Apr 3	37000	30800
1968	Mar 21	14600	7010
1969	Apr 19	23800	24400
1970	Apr 15	17900	8830
1971	Apr 13	21400	13300
1972	Apr 19	32400	30100
1973	Mar 12	28200	17300
1974	May 17	55200	54600
1975	Apr 19	58700	30100
1976	Mar 21	42800	20900
1977	Mar 13	40400	20100
1978	Apr 12	26600	16400
1979	Apr 14	36000	27900

Flows printed in bold are estimated based on daily mean flows.
* Flow is not maximum observed in this year.

TABLE B-2 MAXIMUM NATURAL FLOW VERSUS CORRESPONDING OBSERVED FLOW

GRAND RIVER AT BRANTFORD

YEAR	DATE	MAXIMUM NATURAL FLOW PEAK (cfs)	CORRESPONDING OBSERVED FLOW PEAK (cfs)
1948	Mar 20	**61100**	-----
1949	Mar 23	**29800**	40000
1950	Apr 5	**53100**	25700
1951	Mar 31	**36200**	12900
1952	Apr 2	**25500**	20800
1953	May 3	**18600**	17200 *
1954	Apr 17	**38300**	39000 *
1955	Mar 12	26000	25800
1956	Apr 5	40500	40300
1957	Feb 27	**20300**	20600 *
1958	Mar 29	**11300**	6270
1959	Apr 6	35700	25800
1960	Apr 4	55500	40600
1961	Feb 25	14100	14600
1962	Mar 30	34600	19100
1963	Mar 28	36900	24300
1964	Apr 8	17000	7900 *
1965	Apr 8	33600	29000 *
1966	Mar 19	9320	2960
1967	Apr 3	36500	34900
1968	Mar 21	18400	12700 *
1969	Apr 19	24300	25800
1970	Apr 15	17500	12800
1971	Apr 13	24300	16500
1972	Apr 19	38000	35700
1973	Mar 12	33800	23700
1974	May 17	64700	63000
1975	Apr 19	64000	37200
1976	Mar 21	52200	28200
1977	Mar 13	53200	32400
1978	Apr 12	33100	21100
1979	Apr 14	52700	46400

Flows printed in bold are estimated based on daily mean flows.
* Flow is not maximum observed in this year.

FIGURE B-3 MAXIMUM NATURAL FLOW VERSUS CORRESPONDING OBSERVED FLOW

SPEED RIVER BELOW GUELPH

YEAR	DATE	MAXIMUM NATURAL FLOW PEAK (cfs)	CORRESPONDING OBSERVED FLOW PEAK (cfs)
1948		-----	
1949	Mar 23	-----	
1950	Apr 5	6720	
1951	Mar 31	3780	
1952	Apr 2	**1870**	
1953	May 3	**1660**	
1954	Apr 17	**2870**	
1955	Mar 12	**2420**	
1956	Apr 5	**3730**	
1957	Feb 27	**1460**	
1958	Mar 29	**712**	
1959	Apr 6	**3410**	
1960	Apr 4	3010	
1961	Feb 25	**1430**	
1962	Mar 30	2170	
1963	Mar 28	3000	
1964	Apr 8	1020	
1965	Apr 8	3460	
1966	Mar 19	920	
1967	Apr 3	4810	
1968	Mar 21	2300	
1969	Apr 19	2750	
1970	Apr 15	1640	
1971	Apr 13	1700	
1972	Apr 19	3730	
1973	Mar 12	2930	
1974	May 17	5850	
1975	Apr 19	4480	
1976	Mar 21	3500	1770
1977	Mar 13	3700	1830
1978	Apr 12	2280	1220
1979	Apr 14	3340	2020

Flows printed in bold are estimated based on daily mean flows.
* Flow is not maximum observed in this year.

APPENDIX C

FREQUENCY DISTRIBUTIONS

Appendix C

FREQUENCY DISTRIBUTIONS

Excerpted from "Flood Damage Reduction Program Flood Frequency Analysis", R. Condie, G.A. Nix, and L.G. Boone, Environment Canada, January 1977.

This appendix gives a brief mathematical summary of the frequency distributions used in the program. Only those features of the distributions that are necessary for an understanding of the program are given, along with the solution. Four frequency distributions are considered, the Gumbel I, the Log-Normal, the Three Parameter Log-Normal and the Log-Pearson III, and as far as possible they are fitted to the flood data by the method of maximum likelihood.

For the Three Parameter Log-Normal with the probability density function in its conventional form, there will seldom be a maximum likelihood fit when the untransformed data exhibit negative skewness and in such cases there can never be a moment fit either. On the other hand, a maximum likelihood fit may not be obtained when the data have a very small positive skewness but a moment fit is always possible. In those cases, the program switches to a moment fit and identifies it as such on the output.

The Log-Pearson III will be bounded above when the logarithms of the flood data have negative skewness; when positive the distribution will be bounded below. In general, for Canadian rivers, this distribution will be bounded above and it is difficult to interpret the analysis. Sometimes the maximum likelihood fit gives results which seem, subjectively, better than those obtained by moments, and vice versa. For this distribution the frequency regime is given for both maximum likelihood and moment fitting.

GENERAL:

Population Statistics

Given a sample x_1, x_2, ... xN, the best estimates of the population mean, standard deviation, and skew and kurtosis coefficients as computed herein are given by

Mean, $x = (1/N)\Sigma x$ G(1)

Standard Deviation, $S = \{[1/(N-1)]\Sigma(x-\bar{x})^2\}^{1/2}$ G(2)

Skew Coefficient, $\beta_1 = \{N^2/[(N-1)(N-2)]\}(m_3/S^3)$ G(3)

Coefficient of Kurtosis, $\beta_2 = \{[N^2(N+1)]/[(N-1)(N-2)(N-3)]\}(m_4/S^4)$ G(4)

where m_3 and m_4, the third and fourth central moments are defined by

$m_3 = (1/N)\Sigma(x-\bar{x})^3$ G(5)

and $m_4 = (1/N)\Sigma(x-\bar{x})^4$ G(6)

The summations are carried out over the N terms of the data series. When working in logarithmic units, the same definitions apply with x replaced by its natural logarithm, lnx. There is no universally accepted system of statistical notation and for that reason symbols are identified throughout the text. Distribution parameters when estimated from a sample are distinguished by putting a circumflex over them.

The Cumultive Probability Function or General Frequency Equation

This function gives the probability of an event less than a stated magnitude. For most frequency distributions it cannot be expressed in closed form. In this program which deals specifically with flood frequency analysis it is rearranged to give the T year flood as a function of the distribution parameters only, or a function of these parameters and the standard normal deviate t.

Maximum Likelihood Estimators

For a given probability density function ϕ (x; a, b, c, ...), where a, b, c, etc. are parameters, the joint probability that a sample of N values x_1, x_2,...xN came from that distribution is

$$L = \phi(x_1;\ a,\ b,\ c,\ \ldots)\phi(x_2;\ a,\ b,\ c,\ \ldots)\ \ldots\ \phi(x_N;\ a,\ b,\ c,\ \ldots)$$

L is the likelihood and the principle of maximum likelihood states that the values of a, b, c, etc. should be chosen to maximize L. When dealing with continuous distributions it usually becomes simpler to maximize the natural logarithm of the likelihood function lnL.

The system of equations which when solved achieve this maximization are the Maximum Likelihood Estimators for the parameters of the distribution, and the system is

$$\partial \ln L/\partial a = \partial \ln L/\partial b = \partial \ln L/\partial c \text{ etc.} = 0$$

Minimum variance estimates of parameters are obtained by this method.

The Asymptotic Standard Error of Estimate

If z is a function of say three statistical variables a, b and c, each subject to sampling variance, then the variance of z is

$$\begin{aligned} \mathrm{var}(z) = {} & (\partial z/\partial a)^2\mathrm{var}(a) + (\partial z/\partial b)^2\mathrm{var}(b) + (\partial z/\partial c)^2\mathrm{var}(c) \\ & + 2(\partial z/\partial a)(\partial z/\partial b)\mathrm{cov}(a,b) + 2(\partial z/\partial a)(\partial z/\partial c)\mathrm{cov}(a,c) \\ & + 2(\partial z/\partial b)(\partial z/\partial c)\mathrm{cov}(b,c) \qquad \text{G(5)} \end{aligned}$$

Equation G(5) is the general variance of estimate equation and is applicable to a function of any number of variables. The required partial derivatives are obtained from the general frequency equation and are then evaluated at the estimated values $\hat{a}$, $\hat{b}$, $\hat{c}$. The required variances and covariances are obtained from the inverse dispersion matrix using the estimated values $\hat{a}$, $\hat{b}$, $\hat{c}$. Sometimes, particularly for two parameter distributions Equation G(5) reduces to a relatively simple function and the Asymptotic Standard Error of Estimate is root var(z). For the three parameter distributions the expression is usually too complex to be shown within the confines of a single page. In this text for the Three Parameter Log-Normal and Log-Pearson Type III distributions only the required partial derivatives and the inverse dispersion matrix are given. Matrix inversion to give the required variances and covariances is performed by a subroutine in the program.

THE DISTRIBUTIONS USED

A. THE GUMBEL I DISTRIBUTION:

Fisher and Tippett's first asymptotic distribution of extreme values, sometimes known as the double expotential distribution, was extensively investigated by E.J. Gumbel and proposed for use in flood frequency analysis. In view of his many contributions in this field it is usually known in hydrology as the Gumbel I distribution.

It has two parameters, scale and location, the variate is unbounded either above or below, and has fixed coefficients of skew and kurtosis of 1.14 and 5.4 respectively. Because of its simplicity and also having some theoretical justification, use of this distribution was formerly much favoured in flood frequency analysis. The lack of a lower boundary can at times lead to negative floods at shorter return periods, a condition which is not physically possible. On the assumption that the probability of occurrence of less than zero is the probability mass of zero flood, the implicaton is that the river may be dry for an entire year, if it is the annual flood series which is being analysed.

By considering these few simple characteristics, a reasonable assessment can be made of how well this distribution has been fitted to the flood data being analysed. The occurrence in the analysis of negative values at shorter return periods should be examined jointly with the plotted data and of the population coefficients of skewness and kurtosis as computed from the sample, and if their values are fairly near the theoretical values of 1.14 and 5.4 it can reasonably be concluded that a good fit has been obtained.

The Probability Density Function

$$\phi(x) = ae^{-a(x-u)}e^{-e^{-a(x-u)}} \qquad A(1)$$

The Cumulative Probability Function

$$P = e^{-c^{-a(x-u)}} \qquad A(2)$$

where P is the probability of a value less than x, a is a scale parameter, a > o, and u is a location parameter. These parameters are estimated from the sample using maximum likelihood theory. To distinguish these estimated parameters they are designated a and u.

Maximum Likelihood Estimators

Applying maximum likelihood theory to the p.d.f. at Equation A(1) gives the following

$$\partial \ln L/\partial a = N/a - \Sigma(x-u) + \Sigma\{(x-u)\exp. - a(x-u)\} = 0 \qquad A(3)$$

$$\partial \ln L/\partial u = Na - a\Sigma\{\exp. - a(x-u)\} = 0 \qquad A(4)$$

where the summations are taken over the N terms of the sample.

Re-arranging Equations A(3) and A(4) gives.

$$a\Sigma xe^{-ax} - (a\bar{x}-1)\Sigma e^{-ax} = 0 \qquad A(5)$$

$$u = (1/a)(\ln N - \ln\Sigma e^{-ax}) \qquad A(6)$$

Equations A(5) and A(6) were given by Kimball, Reference (5). The curve corresponding to Equation A(5) has two inflexions and is solved by using the Bolzano method first to isolate the root clear of the inflexions, then by the Newton-Raphson method. The estimated parameter a is then substituted in Equation A(6) to give $\hat{u}$.

The Frequency Regime

Re-arranging Equation A(2) and replacing a and u by their estimates, $\hat{a}$, $\hat{u}$, gives

$$x_t = \hat{u} - (1/\hat{a})\ln.\ln\{T/(T-1)\} \qquad A(7)$$

where x_t is the value of the variate corresponding to a return period of T years.

The Asymptotic Standard Error of Estimate

The inverse dispersion matrix can be derived from Equations A(3) and A(4). Inverting, simplifying, and substituting in the general equation for the variance of estimate gives

$$\mathrm{var}(x_t) = \{1/(a^2N)\}\{1+(1-\gamma+y_t)^2/(\Pi^2/6)\} \qquad A(8)$$

where γ is Euler's constant, approximately 0.5772 and y_t is the reduced variate $a(x_t-u)$.

This expression is given by E.J. Gumbel, Reference (6) following the work of B.F. Kimball, Reference (7). Replacing a and u by their estimates $\hat{a}$ and $\hat{u}$, and taking the square root of $\mathrm{var}(x_t)$ gives the Asymptotic Standard Error of the T year event.

B. THE LOG-NORMAL DISTRIBUTION:

Hydrologic events can seldom be described by the normal distribution since these events are more commonly skewed. Many theoretical transformations have been developed for normalizing skewed distributions, because it is generally easier to draw statistical conclusions from the normal distribution whose exact theoretical properties are well known. In flood frequency analysis it is generally necessary to extrapolate far beyond the range of observed events and the idea of a normalizing transformation is very attractive. One such transformation leads to the Log-Normal distribution. This distribution has been around for quite a long time. Galton noted in 1875 that the geometric mean of some types of data series was a better descriptor than the arithmetic mean. The geometric mean is of course the arithmetic mean of the logarithms, or rather the antilog of such, and hence the name Log-Normal. Some older text books still refer to this distribution as the Galton distribution. Hazen in 1930 applied this distribution to flood frequency analysis and the distribution is known by three names; the Log-Normal, Hazen, or Galton distributions. Essentially use of this distribution implies that the logarithms of the basic data to be analysis are normally distributed.

It is a simple two parameter distribution and the two parameters jointly influence the location of the mean, the scale, and the shape. The variate is bounded below at zero and has no upper boundary, and this distribution cannot have negative skewness. This latter feature is very important because the Log-Normal distribution can be fitted to negatively skewed data but the analysis will be grossly in error. By virtue of the logatithmic transform the theoretical coefficients of skew and kurtosis of the transformed data are respectively 0.0 and 3.0.

These theoretical characteristics give a handy subjective assessment of the validity of fitting the Log-Normal distribution to the data sample of annual floods. The program output gives best estimates of the coefficients of skew and kurtosis of the logarithms of the population as estimated from the available sample of annual floods. If their values do not vary very much from the theoretical values of 0.0 and 3.0, then the distribution will give a fairly good estimate of the flood frequency regime at the particular location. If the skewness of the logarithms is substantially positive then floods at the longer return periods will be underestimated and if negative, these floods will be over estimated. These conditions can be verified by examining the plotted data.

Experience with Canadian rivers shows that the logarithmic transform of the annual flood series generally results in overdoing the normalizing process. Basic data which are positively skewed show sometimes a quite substantially negative skew coefficient of the logarithmic transform.

The Probability Density Function

$$\phi(x) = \frac{1}{x\sigma\sqrt{2\Pi}} \exp. - \frac{(\ln x - m)^2}{2\sigma^2} \qquad B(1)$$

where m and o^2 are respectively location and scale parameters of the transformed variate lnx. For the untransformed variate they jointly influence the skewness, but the lower boundary is fixed at zero.

The Cumulative Probability Function

The cumulative probability function cannot be expressed in closed form and instead, the general frequency equation is used

$$x_t = \exp.(m+t\sigma) \qquad B(2)$$

The parameters are replaced by their estimates $\hat{m}$ and $\hat{\sigma}$ and $\hat{t}$ is the standard normal deviate at the required return period.

Maximum Likelihood Estimators

Applying maximum likelihood theory to the p.d.f. at Equation B(1) gives

$$\partial \ln L/\partial m = -(Nm)/\sigma^2 + 1/\sigma^2 \Sigma \ln x = 0 \qquad B(3)$$

$$\partial \ln L/\partial(\sigma^2) = -N/(2\sigma^2) + 1/(2\sigma^4)\Sigma(\ln x - m)^2 = 0 \qquad B(4)$$

Re-arranging Equations B(3) and B(4) gives

$$m = (1/N)\Sigma \ln x \qquad B(5)$$

$$\sigma^2 = (1/N)\Sigma(\ln x - m)^2 \qquad B(6)$$

Equation B(6) gives a biassed estimate of o^2 and is corrected for the bias to give

$$\sigma^2 = \{1/(N-1)\}\Sigma(\ln x - m)^2 \qquad B(7)$$

Equation B(5) and B(7) give minimum variance unbiassed estimtes $\hat{m}$ and o^2 and m and o.

The Asymptotic Standard Error of Estimate

The inverse dispersion matrix can be derived from Equations B(3) and B(4). Inverting, simplifying, and substituting in the general equation for the variance of estimate gives

$$\mathrm{var}(x_t) = x_t^2(\sigma^2/N)(1+t^2/2) \qquad B(8)$$

Replacing o by its estimte $\hat{\sigma}$ and taking the square root give the Asymptotic Standard Error of Estimate of the T year event.

C. THE THREE PARAMETER LOG-NORMAL DISTRIBUTION:

The success of any normalizing transformation can be measured by computing the coefficients of skew and kurtosis of the transformed data, and the closer these coefficients are to 0.0 and 3.0, the more successful the transformation is. The straightforward transformation used in the Log-Normal distribution is not markedly successful in this respect and it has been found that the inclusion of a third parameter will greatly improve the transformation, hence the name, the Three Parameter Log-Normal distribution. In its most simple terms, the function $y = \ln(x-a)$ is distributed normally.

The three parameters of the distribution jointly influence the location of the mean, the scale and the shape. The variate is bounded below by the parameter a, and has no upper boundary and in its conventional form the distribution cannot have negative skewness. When developing this program some theoretical work was done on the negatiely skewed Three Parameter Log-Normal distribution. The variate in that distribution is unbounded below and has a positive upper boundary at the parameter a. Although this form of the distribution was successfully programmed, it was generally found that the upper boundary parameter was very close to the maximum observed flood in the data series. Subjectively, this form of the distribution should not be used for flood frequency analysis. Further development work compared the results obtained by fitting the distribution by moments with those obtained by a maximum likelihood fit. The maximum likelihood fit was clearly superior in that it gave a smaller standard error of estimate of the T-year event, and also the computed coefficient of skew of the transformed data was much closer to the theoretical value of 0.0. For these reasons, only the maximum likelihood fit is used, except in the circumstances mentioned earlier, when a moment fit is used.

The Three Parameter Log-Normal is an exceptionally flexible distribution and very well suited to flood frequency analysis. When the parameter a becomes zero, then the distribution becomes the straightforward Log-Normal, and it can also be shown theoretically that the Gumbel I distribution is a special case of the Three-Parameter Log-Normal. Extensive testing during the development of this program showed that the coefficient of skew of the transformed data averaged 0.055, with a low value of 0.0008, an excellent check on the success of the fit.

Like the Log-Normal, the validity of fitting the Three Parameter Log-Normal distribution can easly be assessed by comparing the computed values of the skew and kurtosis coefficients with their theoretical values. One minor drawback is that where the Log-Normal distribution shows skewness of the transformed data very close to 0.0, the Three Parameter Log-Normal will usually improve on this but at the expense of an increased standard error of estimate. This is caused by the introduction of the third parameter which is itself subject to sampling variance and is reflected in the overall standard error of estimate. This is not very serious, however, since the T year event obtained from either of these distributions in such cases is pretty well the same.

The Probability Density Function

$$\phi(x) = \frac{1}{(x-a)\sigma\sqrt{2\Pi}} \exp. - \frac{\{\ln(x-a) - m\}^2}{2\sigma^2} \qquad C(1)$$

where m and o^2 are respectively location and scale parameters for the transformed variate ln(x-a). For the untransformed variate they jointly influence the skewness and a is the lower boundary of the variate x.

The Cumulative Probability Fucntion

Like the log-normal distribution, the cumulative probability function cannot be expressed in closed form and the general frequency equation is

$$x_t = a + \exp(m+t\sigma) \qquad C(2)$$

The parameters are replaced by their estimates $\hat{a}$, $\hat{m}$ and $\hat{\sigma}$ and t is the standard normal deviate at the required return period.

Maximum Likelihood Estimators

Applying maximum likelihood theory to the p.d.f. at Equation C(1) gives

$$\partial \ln L/\partial a = (1-m/\sigma^2)\Sigma(x-a)^{-1} + (1/\sigma^2)\Sigma\{[\ln(x-a)](x-a)^{-1}\} = 0 \qquad C(2)$$

$$\partial \ln L/\partial m = (1/\sigma^2)\Sigma\{\ln(x-a)-m\} = 0 \qquad C(3)$$

$$\partial \ln L/\partial(\sigma^2) = -N/(2\sigma^2) + 1/(2\sigma^4)\Sigma\{\ln(x-a)-m\}^2 = 0 \qquad C(4)$$

Equations C(2), and C(3) and C(4) can be re-arranged to give an equation in a only f(a) =

$$\Sigma(x-a)^{-1}\{(1/N)\Sigma\ln^2(x-a) - [(1/N)\Sigma\ln(x-a)]^2 - (1/N)\Sigma\ln(x-a)\}$$
$$+ \Sigma\{\ln(x-a)\}(x-a)^{-1} = 0 \qquad C(5)$$

When developing a solution for Equation C(5) it was noticed that f(a) and f (a) were negative near x min., with both increasing as a decreased. A Newton-Raphson solution was used with iteration starting at 0.8x min. and x min. To avoid missing such a root, f(a) is evaluated at these points and checked for a sign change, and a Bolzano method is included in the program to isolate any such roots.

Having found a, then

$$m = (1/N)\Sigma\ln(x-a) \qquad C(6)$$

and, correcting for bias

$$\sigma^2 = \{1/(N-1)\}\Sigma\{\ln(x-a)-m\}^2 \qquad (7)$$

Hence the estimated parameters $\hat{a}$, $\hat{m}$ and $\hat{\sigma}^2$.

The Asymptotic Standard Error of Estimate

For the general variance of estimate equation, the required partial derivatives are

$$\partial x_t/\partial a = 1 \qquad C(8)$$

$$\partial x_t/\partial m = \exp.(m+t\sigma) \qquad C(9)$$

$$\partial x_t/\partial m^2 = (1/2\sigma^2)\exp.(m+t\sigma) \qquad C(10)$$

and the inverse dispersion matrix V^{-1} is

$$N/\sigma^2 \begin{matrix} (1+\sigma^2)\exp.(2\sigma^2-2m) & \exp.(\sigma^2/2-m) & -\exp.(\sigma^2/2-m) \\ \exp.(\sigma^2/2-m) & 1 & 0 \\ -\exp.(\sigma^2/2-m) & 0 & 1/2\sigma^2 \end{matrix} \qquad C(11)$$

Equations C(8) through C(10) are evaluated at the estimated values $\hat{m}$ and $\hat{\sigma}$ with the appropriate t for the required probability level. The elements of the matrix are evaluated at $\hat{m}$ and $\hat{\sigma}^2$ and inverted to give the variances and covariances. Substitution in the general variance of estimate equation and taking the square root give the Asymptotic Standard Error of Estimate of the T year event.

Fitting by Moments

Taking moments of the p.d.f. given by equation C(1), re-arranging and replacing them by their sample estimates gives

$$k^3 + 3k - \hat{g}_1 = 0 \qquad C(12)$$

$$\hat{a} = \bar{x} - \hat{s}/k \qquad C(13)$$

$$\hat{m} = \ln(\hat{s}/k) - 1/2\ln(k^2+1) \qquad C(14)$$

$$\sigma^2 = \ln(k^2+1) \qquad C(15)$$

Where s and g_1 are computed from Equations G(2) and G(3).

For positive skew and from Descartes' rule of signs Equation C(12) has only one real root and it is positive. The cubic is solved directly for k, then $\hat{a}$, $\hat{m}$ and $\hat{\sigma}$ follow. For negative skew, Equation C(12) has only one real root, which is negative. Equation C(14) therefore cannot be evaluated.

The Asymptotic Standard Error for Estimate of the T year event is computed according to Kite, Reference (8). The method is quoted here practically in full. Parameterization is slightly different to avoid confusion. The T year event can be expressed in terms of the first moment about the origin, the second and third central moments of the population and the standard normal deviate t. Denoting these moments by u_1, u_2, u_3 and the population skew coefficient by .

$$x_t = z = \mu_1' + F\mu_2^{1/2} \qquad C(16)$$

$$\text{where } F = \frac{\exp.\{[\ln(1+k^2)]^{1/2}t - [\ln(1+k^2)]/2\} - 1\cdot 0}{k} \qquad C(17)$$

and k is the root of Equation C(12).

The direct solution of Equation C(12) is

$$k = (1-w^{2/3})/w^{1/3} \qquad C(18)$$

where $w = \{(\gamma_1{}^2+4)^{1/2} - \gamma_1\}/2$ C(19)

Since $\gamma_1 = u_3/u_2{}^{3/2}$, Equation C(16) can be rewritten

$$x_t = z = f(u_1, u_2, u_3, t)$$

t is invariable and from Equation G(5)

$$\begin{aligned} \mathrm{var}(z) = {} & (\partial z/\partial\mu_1')^2\mathrm{var}(\mu_1') + (\partial z/\partial\mu_2)^2\mathrm{var}(\mu_2) + (\partial z/\partial\mu_3)\mathrm{var}(\mu_3) \\ & + 2(\partial z/\partial\mu_1')(\partial z/\partial\mu_2)\mathrm{cov}(\mu_1',\mu_2) + 2(\partial z/\partial\mu_1')(\partial z/\partial\mu_3)\mathrm{cov}(\mu_1',\mu_3) \\ & + 2(\partial z/\partial\mu_2)(\partial z/\partial\mu_3)\mathrm{cov}(\mu_2,\mu_3) \qquad C(20) \end{aligned}$$

The required partial derivatives are

$$\partial z/\partial\mu_1' = 1 \qquad C(21)$$

$$\partial z/\partial\mu_2 = (1/2\mu_2{}^{1/2})(F-3\gamma_1\partial F/\partial\gamma_1) \qquad C(22)$$

$$\partial z/\partial\mu_3 = (1/\mu_2)(\partial F/\partial\gamma_1) \qquad C(23)$$

F has been defined at Equation C(17) and

$$\partial F/\partial\gamma_1 = (\partial F/\partial k)(\partial k/\partial w)(\partial w/\partial\gamma_1) \qquad C(24)$$

with

$$\begin{aligned} \partial F/\partial k = & \left\{\frac{2k}{1+k^2}\right\}\left\{\left[\exp.\{[\ln(1+k^2)]t^{1/2} - [\ln(1+k^2)]/2\}\right]\cdot \right. \\ & \left. \left[\frac{t}{2k[\ln(1+k^2)]^{1/2}} - \frac{1}{k} - \frac{1}{2k^3}\right] + \frac{1}{2k^3} + \frac{1}{2k}\right\} \qquad C(25) \end{aligned}$$

$$\partial k/\partial w = -\frac{1}{3w^{2/3}}\left[1 + \frac{1}{w^{2/3}}\right] \qquad C(26)$$

$$\partial w/\partial\gamma_1 = -\frac{1}{2} + \left[\frac{\gamma_1}{2(\gamma_1{}^2+4)^{1/2}}\right] \qquad C(27)$$

The computed values of w and k, and the appropriate t for the required probability level, and replacing γ_1 by its sample estimate $\hat{g}_1$ will then substituted in Equations C(25) through C(27) give $\partial F/\partial\gamma_1$ from Equation C (24). Substituting for F and $\partial F/\partial\gamma_1$ and replacing u_2 by its sample estimate $\hat{s}^2$ in Equations C(21) through C(23) give the required partial derivatives from Equation C(20).

The required variances and covariances are

$$\text{var } \mu_1' = \mu_2/N \qquad \text{C(28)}$$

$$\text{var } \mu_2 = \frac{1}{N}(\mu_4 - \mu_2^2) \qquad \text{C(29)}$$

$$\text{var } \mu_3 = \frac{1}{N}(\mu_6 - \mu_3{}^2 - 6\mu_4\mu_2 + 9\mu_2^3) \qquad \text{C(30)}$$

$$\text{cov } (\mu_1', \mu_2) = \mu_3/N \qquad \text{C(31)}$$

$$\text{cov } (\mu_1', \mu_3) = \frac{1}{N}(\mu_4 - 3\mu_2^2) \qquad \text{C(32)}$$

$$\text{cov } (\mu_2, \mu_3) = \frac{1}{N}(\mu_5 - 4\mu_3\mu_2) \qquad \text{C(33)}$$

For the Three Parameter Log-Normal distribution the population central moments u_2 through u_6 are defined by

$$\mu_2 = e^{\sigma^2+2} \ (e^{\sigma^2} - 1) \qquad \text{C(34)}$$

$$\mu_3 = e^{3\sigma^2/2+3} \ (e^{\sigma^2} - 1)^2 \ (e^{\sigma^2} + 2) \qquad \text{C(35)}$$

$$\mu_4 = e^{2\sigma^2+4} \ (e^{\sigma^2} - 1)^2 \ (e^{4\sigma^2} + 2e^{3\sigma^2} + 3e^{2\sigma^2} - 3) \qquad \text{C(36)}$$

$$\mu_5 = e^{5\sigma^2/2+5} \ (e^{10\sigma^2} - 5e^{6\sigma^2} + 10e^{3\sigma^2} - 10e^{\sigma^2} + 4) \qquad \text{C(37)}$$

$$\mu_6 = e^{3\sigma^2+6} \ (e^{15\sigma^2} - 6e^{10\sigma^2} + 15e^{6\sigma^2} - 20e^{3\sigma^2} + 15e^{\sigma^2} - 5) \qquad \text{C(38)}$$

Replacing σ^2 by its sample estimate $\hat{s}^2$ in Equations C(34) through C(38) gives the central moments μ_2 through μ_6 and hence the variances and covariances from Equations C(28) through C(33). Substitution in Equation C(20) gives the variance of the T year event. Finally taking the square root gives the Asymptotic Standard Error. Occasionally this variance may be negative and the Asymptotic Standard Error is printed out as zeroes.

D. THE LOG-PEARSON TYPE III DISTRIBUTION

A group of frequency distributions can be derived from a generalized differential equation proposed by Karl Pearson. This generalized equation has four constants and by equating some to zero or to each other and solving the differential equation a series of symetrical or skewed distributions are found. One solutions leads to a two-parameter distribution and by combinations of these parameters a variety of U and J-shaped distributions are obtainable. Reverting to the generalized differential equation, a possible solution leads to Pearson's Type III distribution and if it is assumed that the logarithm of the variate follows a Pearson Type III distribution, the distribution of the variate itself is the Log-Pearson Type III.

In 1967 the U.S. Water Resources Council recommended its use as the standard method of flood frequency analysis with provision for departure when justified. The Council's decision was based on a need for standardization, convenience, since many agencies used it, and on the analysis of flood data from 10 American rivers. In developing this program and using data from 37 long term hydrometric stations on Canadian rivers it is unlikely that the same recommendation would be made.

The Log-Pearson Type III distribution has three parameters and Bobee, Reference (1), shows the many different shapes the distribution can take, depending on the relationship between the parameters and their signs. Not all these shapes seem acceptable in flood frequency analysis and the user of this program is advised to study Reference (4), particularly with regard to the distribution shape. The range of the variate also depends on the sign of one of the parameters. The variate may have a positive lower boundary and be unbounded above, or, it may have a zero lower boundary with a positive upper bound. A positive upper bound was found in about 67 percent of the test stations and, depending on the position of this upper bound relative to the largest observed flood in the data sample, it can be very difficult to judge how well this distribution fits the data. Cases have been found where an upper boundary parameter was found less than the largest observed flood.

The validity of the fit of the Log-Pearson Type III distribution to the flood series cannot be assessed so easily as for the Gumbel I, the Log-Normal or the Three Parameter Log-Normal distributions although some assistance is given to the user from the program output. When combinations of parameters lead to apparently impractical shapes of the distribution, or where an upper bound is found to be less than the greatest observed flood, the flood frequency regime is given in the output as a set of zeros. The distribution parameters will always be given in the output and the user may relate them to the combinations shown in Reference (4). It is very difficult to give any advice on the interpretation of an upper boundary and its implications. Points to be considered are the proximity of the upper boundary to the largest observed flood and the general appearance of the plot. Sometimes fitting by the method of moments appears better than the maximum likelihood fit and for that reason the flood frequency regime is given using both fitting methods.

The Probability Density Function

$$\phi(x) = \frac{\exp\{-(\ln x-m)/a\}\ \{(\ln x-m)/a\}^{b-1}}{|a|x\Gamma(b)} \qquad D(1)$$

where a, b and m are respectively scale, shape and location parameters and Γ is the gamma function of the argument within parenthesis.

Bobée, Reference (4), shows the many different shapes which this p.d.f. can take, depending on the relationship between the parameters, and not all these shapes are credible in flood frequency hydrology. This is particularly so in cases where "a" is negative, and this is generally so for flood series on Canadian rivers.

The Cumulative Probability Function

Again, the cumulative probability function cannot be expressed in closed form. Making the substitution $y = (\ln x - m)/a$ in the p.d.f. at D(1) and using the Wilson-Hifferty approximation for chi-square, Reference (9).

$$\ln x_t = m + a\{t/(3b^{1/6}) - 1/(9b^{2/3}) + b^{1/3}\}^3 \qquad D(2)$$

where $\ln x_t$ is the logarithm of the T year event and t is the standard normal deviate for the required return period. Then $x_t = \exp(\ln x_t)$.

Maximum Likelihood Estimators

Applying maximum likelihood theory to the p.d.f. at Equation D(1) gives

$$\partial \ln L/\partial a = (1/a^2)\Sigma(\ln x-m) - (1/a)Nb = 0 \qquad D(3)$$

$$\partial \ln L/\partial b = -N\psi(b) + \Sigma \ln\{(\ln x-m)/a\} = 0 \qquad D(4)$$

$$\partial \ln L/\partial m = 1/a - (b-1)\Sigma(\ln x-m) = 0 \qquad D(5)$$

where ψ (b) is the Digamma or Psi function, $\partial \ln\Gamma(b)/\partial b$.

Equations (3), (4), and (5) are three simultaneous transcendental equations in a, b, and m, but Equation (4) can be reduced to a single transcendental equation in m by re-arranging Equations (3) and (5) to give

$$a = (1/Nb)\Sigma \ln(x-m) \qquad D(6)$$

$$b = \Sigma(\ln x-m)^{-1}/\{\Sigma(\ln x-m)^{-1} - N^2/\Sigma(\ln x-m)\} \qquad D(7)$$

Substitution of Equation (6) in Equation (4) eliminates a, then substitution of Equation (7) in Equation (4) eliminates b. The Psi function can be replaced by a combined recurrence formulate and asymptotic expansion where (b) tends to

$$\ln(b+2) - \frac{1}{2(b+2)} - \frac{1}{12(b+2)^2} + \frac{1}{120(b+2)^4} - \frac{1}{252(b+2)^6} - \frac{1}{(1+b)} - \frac{1}{b} \qquad D(8)$$

It is quite clearly impossible to show these substitutions within the confines of one page. The distribution at Equation (1) can be either positively or negatively skewed. It can be shown quite easily by considering the moments of the distribution that if the skew coefficient of the logarithms is positive, a must be positive and therefore from Equation (4), (lnx-m) must always be positive for all values of lnx. It follows then that the distribution is bounded below at exp. m. It is more common in flood series from Canadian rivers to find that the skew coefficient of the logarithms is negative, and (lnx-m) must always be negative. Thus the distribution is bounded above at exp. m.

The solution of Equation (4) with the necessary substitutions can therefore have roots either less than the minimum of the data series or greater than the maximum of the data series. The curve corresponding to Equation (4) has several inflexions and for reliability a Bolzano method is used first of all to isolate the root clear of the effect of any inflexions before the faster Newton-Raphson iteration method takes over. This method of solution has so far proved reliable.

The solution then gives the maximum likelihood estimates $\hat{a}$, $\hat{b}$, and $\hat{m}$ for the distribution parameters.

The Asymptotic Standard Error of Estimate

For the general variance of estimate equation the required partial derivatives are, putting $z = \ln x_t$

$$\partial z/\partial a = \{t/3b^{1/6}) - 1/(9b^{2/3}) + b^{1/3}\}^3 \qquad D(9)$$

$$\partial z/\partial b = 3a\{(t/3b^{1/6}) - 1/(9b^{2/3}) + b^{1/3}\}^2\{-t/(18b^{7/6}) + 2/(27b^{5/3}) + 1/(3b^{2/3})\} \qquad D(10)$$

$$\partial z/\partial m = 1 \qquad D(11)$$

and the inverse dispersion matrix V^{-1} is

and the inverse dispersion matrix V^{-1} is

$$V^{-1} = N \begin{matrix} b/a^2 & 1/a & 1/a^2 \\ 1/a & \psi'(b) & 1/\{a(b-1)\} \\ 1/a^2 & 1/\{a(b-1)\} & a^2/(b-2) \end{matrix}$$

where $\psi'(b)$ is the Trigamma Function $\frac{d}{db}\psi(b)$

Equations D (9) and D (10) are evaluated at the estimated values $\hat{a}$ and $\hat{b}$ with the appropriate t for the required return period. The elements of the matrix are evaluated at a and b and inverted to give the required variances and covariances. Substitution in the general variance of estimate equation gives var $(\ln x_t)$. Then, the Asymptotic Standard Error of Estimate of the T year event is

$$\text{S.E. of } x_t = x_t\{\text{var}(\ln x_t)\}^{1/2} \qquad D(12)$$

Fitting by Moments

Taking moments of the p.d.f. given by Equation (1), re-arranging and replacing them with their sample estimates gives

$$\hat{a} = (\hat{s}\hat{g}_1)/2 \qquad D(13)$$

$$\hat{b} = (2/\hat{g}_1)^2 \qquad D(14)$$

$$\hat{m} = \bar{x} - 2\hat{s}/\hat{g}_1 \qquad D(15)$$

where $\bar{x}$, $\hat{s}$ and $\hat{g}_1$ respectively are computed from Equations G(1), G(2) and G(3) with x replaced by its natural logarithm, lnx.

Substitution of $\hat{a}$, $\hat{b}$ and $\hat{m}$ in Equation D(2) and using the appropriate value of t gives $\ln x_t$, and exponentiating gives the T year event.

Bobee, Reference (10) gives the variance of the T year event for a Pearson Type III distribution as a function of the second central moment, the skew coefficient and t. By a simple extension, for the Log-Pearson Type III,

$$\mathrm{var}(\ln x_t) = \frac{\mu_2}{N}\left[1 + K\gamma_1 + \frac{K^2}{2}\left\{\frac{3\gamma_1^2}{4} + 1\right\} + 3K\frac{\partial K}{\partial\gamma_1}\left\{\gamma_1 + \frac{\gamma_1^3}{4}\right\} + 3\left(\frac{\partial K}{\partial\gamma_1}\right)^2\left\{2 + 3\gamma_1^2 + \frac{5\gamma_1^4}{8}\right\}\right] \qquad D(16)$$

where

$$K \approx t + (t^2 - 1)\frac{\gamma_1}{6} + \frac{(t^3 - 6t)}{3}\left(\frac{\gamma_1}{6}\right)^2 - (t^2 - 1)\left(\frac{\gamma_1}{6}\right)^3 + t\left(\frac{\gamma_1}{6}\right)^4 + \frac{1}{3}\left(\frac{\gamma_1}{6}\right)^5 \qquad D(17)$$

and

$$\frac{\partial K}{\partial\gamma_1} \approx \frac{t^2 - 1}{6} + \frac{4(t^3 - 6t)}{6^3}\gamma_1 - \frac{3(t^2 - 1)}{6^3}\gamma_1^2 + \frac{4t}{6^4}\gamma_1^3 - \frac{10}{6^6}\gamma_1^4 \qquad D(18)$$

Replacing μ_2 and γ_1 by their estimates, $\hat{s}^2$ and $\hat{g}_1$ as previously computed for Equations D(13) through D(15), and using the appropriate value of t, the variance of the logarithm of the T year event, $\mathrm{var}(\ln x_t)$, is obtained. Then the Asymptotic Standard Error of Estimate of the T year event follows from

$$\text{S.E. of } x_t = x_t\{\mathrm{var}(\ln x_t)\}^{1/2} \qquad D(19)$$

REFERENCES

(1) Nix, G.A., Condie, R., and Boone, L.G., Program HISTFLO, Flood Frequency Analysis Using Historic Information, Engineering Hydrology Section, Engineering Division, 1975.

(2) Sangal, B.P., and Biswas, A.K., The 3 Parameter Log-Normal Distribution and its Applications in Hydrology, Water Resources Research, 6 (2) pp. 505-515, 1970.

(3) Condie, R., and Kallio, R.W., Computer Programme ML3LN, Frequency Analysis, 3-Parameter Log-Normal Distribution, Parameters by Maximum Likelihood, Engineering Hydrology Section, Engineering Division, 1975.

(4) Bobee, B., The Log Pearson Type 3 Distribution and its Application in Hydrology, Water Resources Research, 11(5), 681-689, 1975.

(5) Kimball, B.F., Sufficient Statistical Estimation Functions for the Parameters of the Distribution of Maximum Values, Annals of Mathematical Statistics, 17, 299, 1946.

(6) Gumbel, E.J., Statistics of Extremes, p. 235, Columbia University Press, New York, 1960.

(7) Kimball, B.F., An Approximation to the Sampling Variances of an Estimated Values of Given Frequency Based on Fit of Doubly Exponential Distribution of Maximum Values, Annals of Mathematical Statistics, 20, 110, 1949.

(8) Kite, G.W., Frequency and Risk Analyses in Hydrology, Internal Document, Chapter 7.4.1, Water Resources Branch, Inland Waters Directorate, Environment Canada, 1976.

(9) Wilson, E.B., and Hilferty, M.M., The Distribution of Chi-square, Proceedings of the National Academy of Science, U.S.A., 17, 684, 1931.

(10) Bobee, B., Sample Error of T-year Events Computed by Fitting a Pearson Type 3 Distribution, Water Resources Research, 9(5), 1264-1270, 1973.

(11) Programming Calcomp Pen Plotters, California Computers Products, Inc., 305 Muller Avenue, Anaheim, California, 92803, 1968.

APPENDIX D

COMPARISON OF FLOOD FREQUENCY USING LOG PEARSON TYPE III AND THREE-PARAMETER LOG NORMAL DISTRIBUTIONS

Figure D-1 GRAND RIVER AT SHAND DAM
Flood Frequency Analysis
Maximum Instantaneous Flow 1948-1978

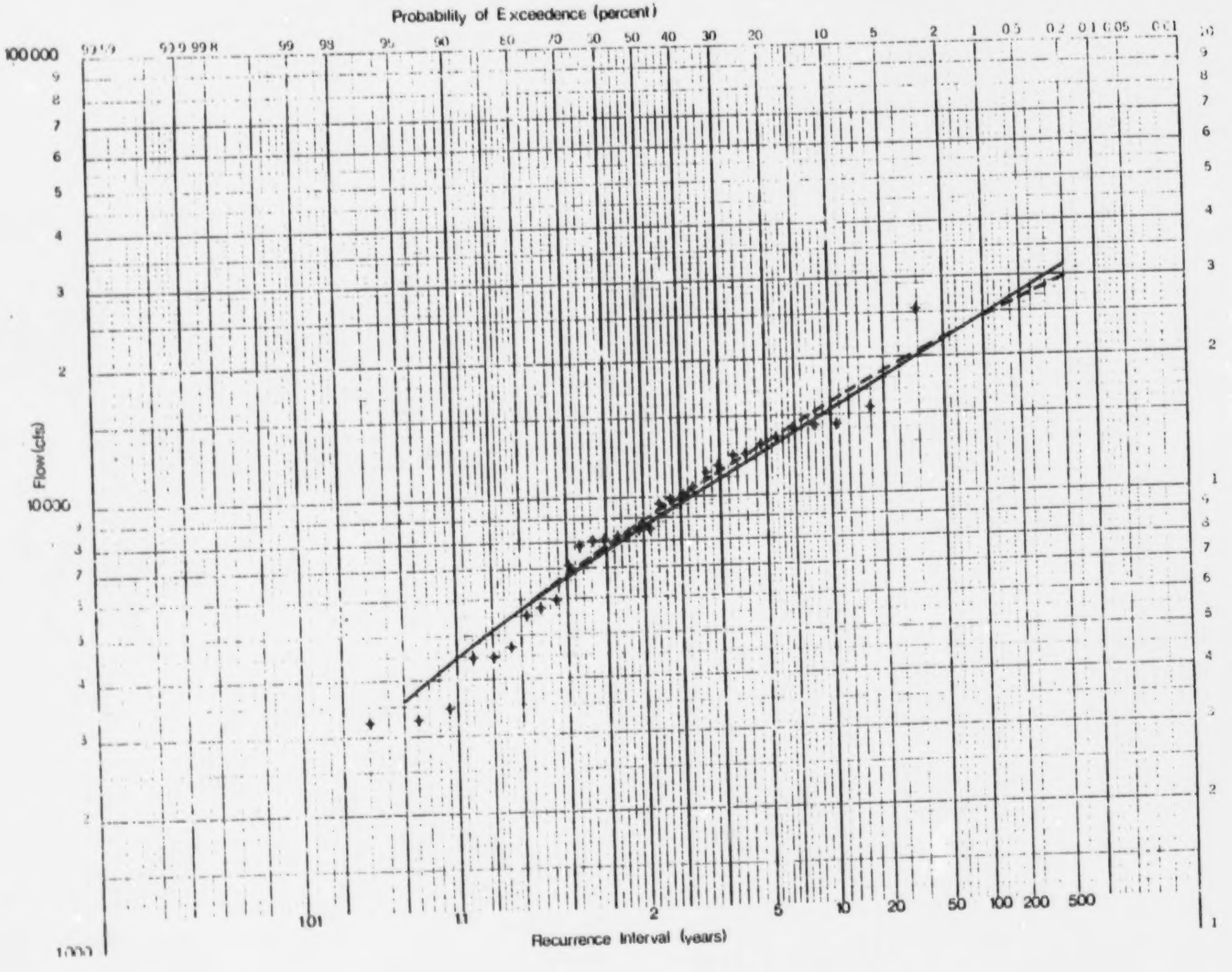


Figure D-2 CONESTOGO RIVER AT CONESTOGO DAM
Flood Frequency Analysis
Maximum Instantaneous Flow 1948-1979

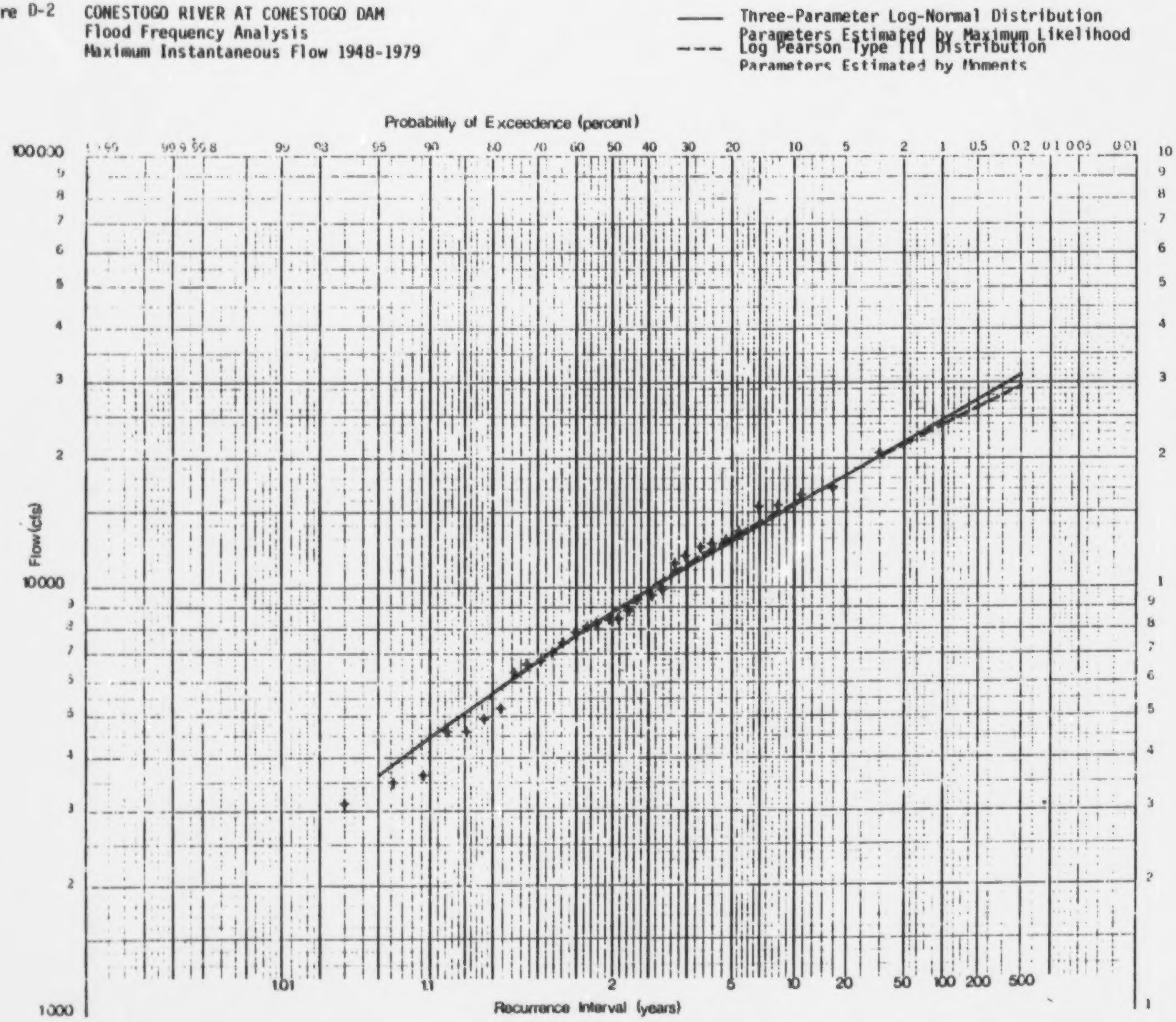


Figure D-3 GRAND RIVER AT GALT
Flood Frequency Analysis
Maximum Instantaneous Flow 1948-1979

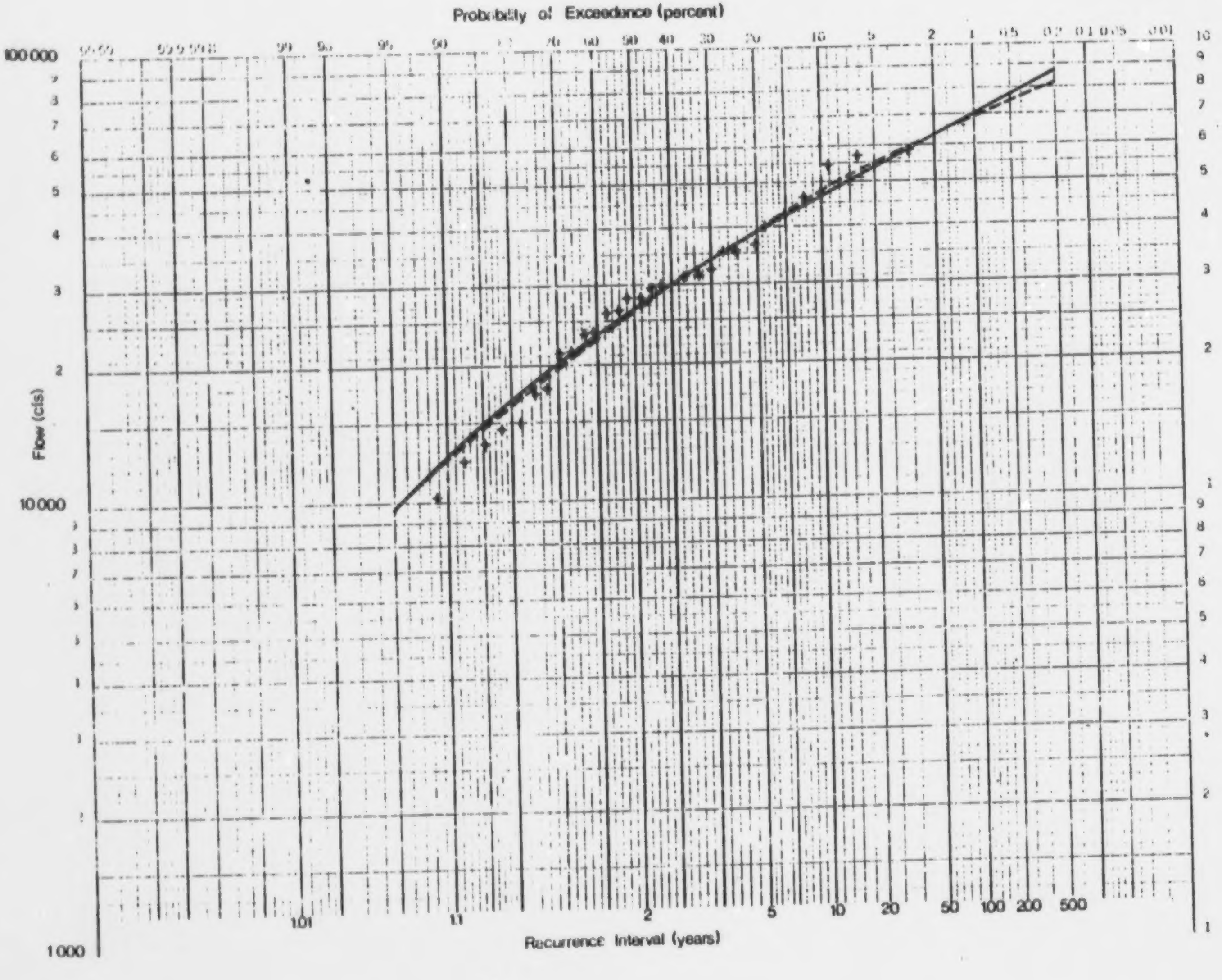


Figure D-4 GRAND RIVER AT BRANTFORD
Flood Frequency Analysis
Maximum Instantaneous Flow 1948-1979

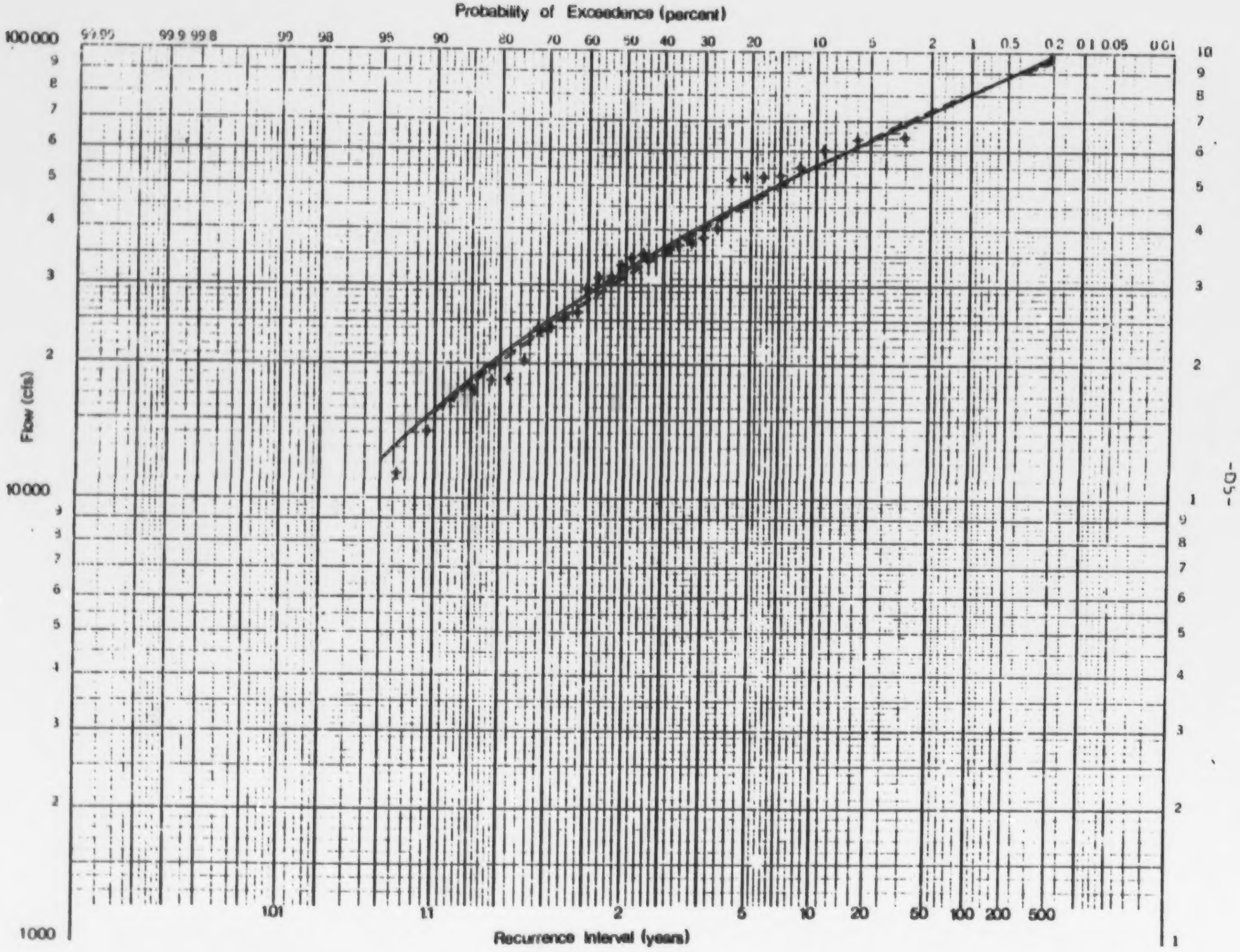


Figure D-5 SPEED RIVER BELOW GUELPH
Flood Frequency Analysis
Maximum Instantaneous Flow 1950-1979

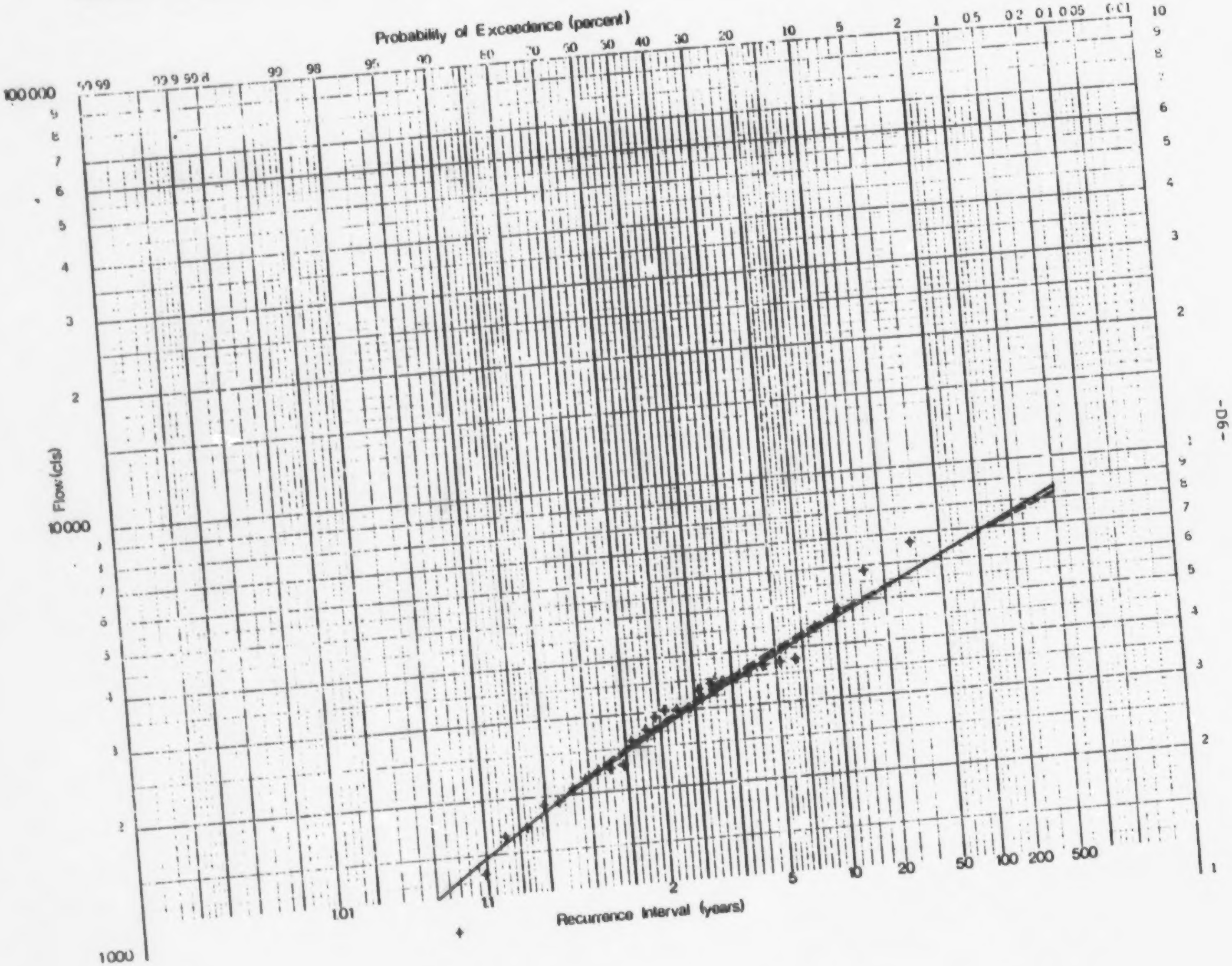


Figure D-6 NITH RIVER AT NEW HAMBURG
Flood Frequency Analysis
Maximum Instantaneous Flow 1951-1979

——— Three-Parameter Log-Normal Distribution
Parameters Estimated by Maximum Likelihood
– – – Log Pearson Type III Distribution
Parameters Estimated by Moments

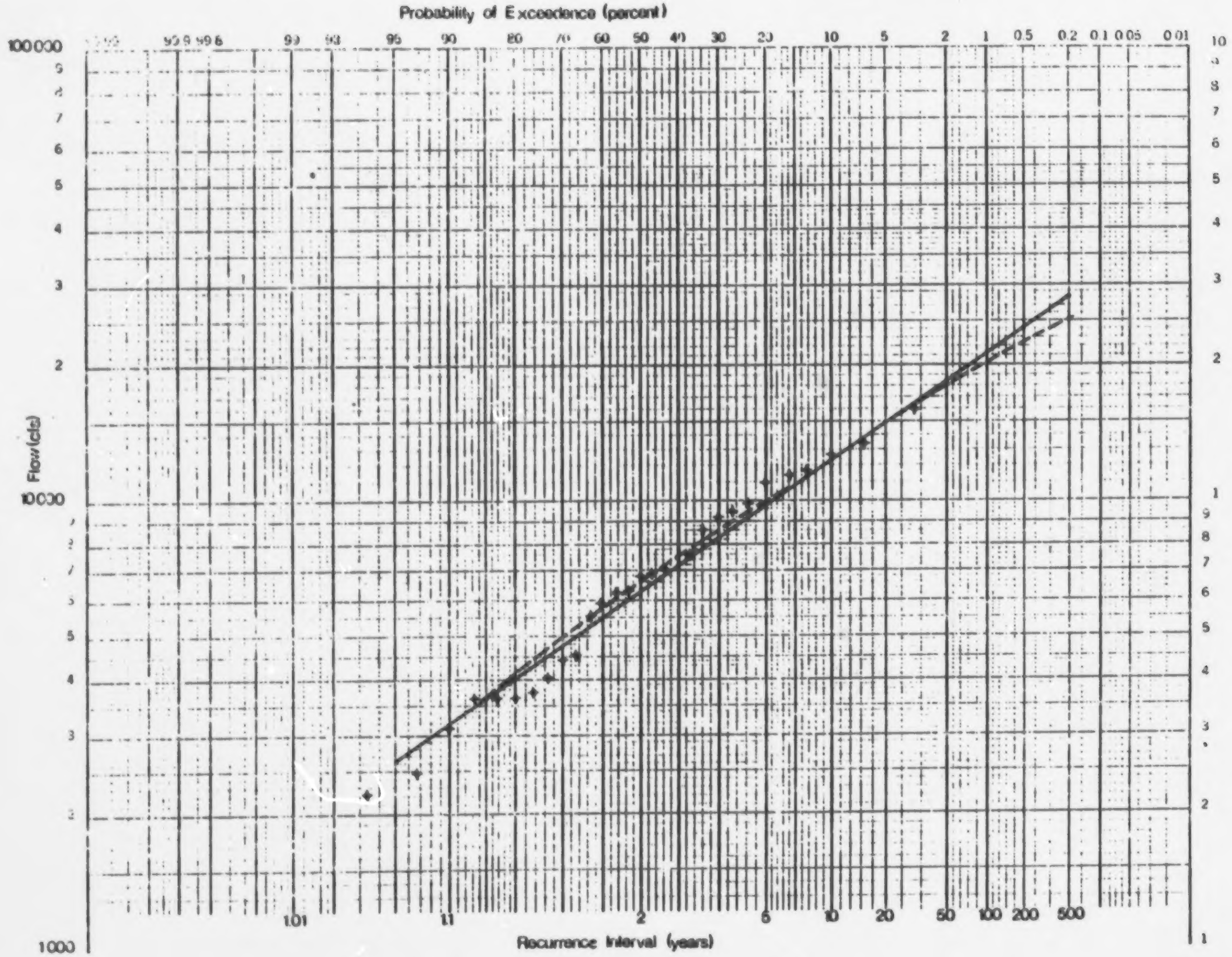
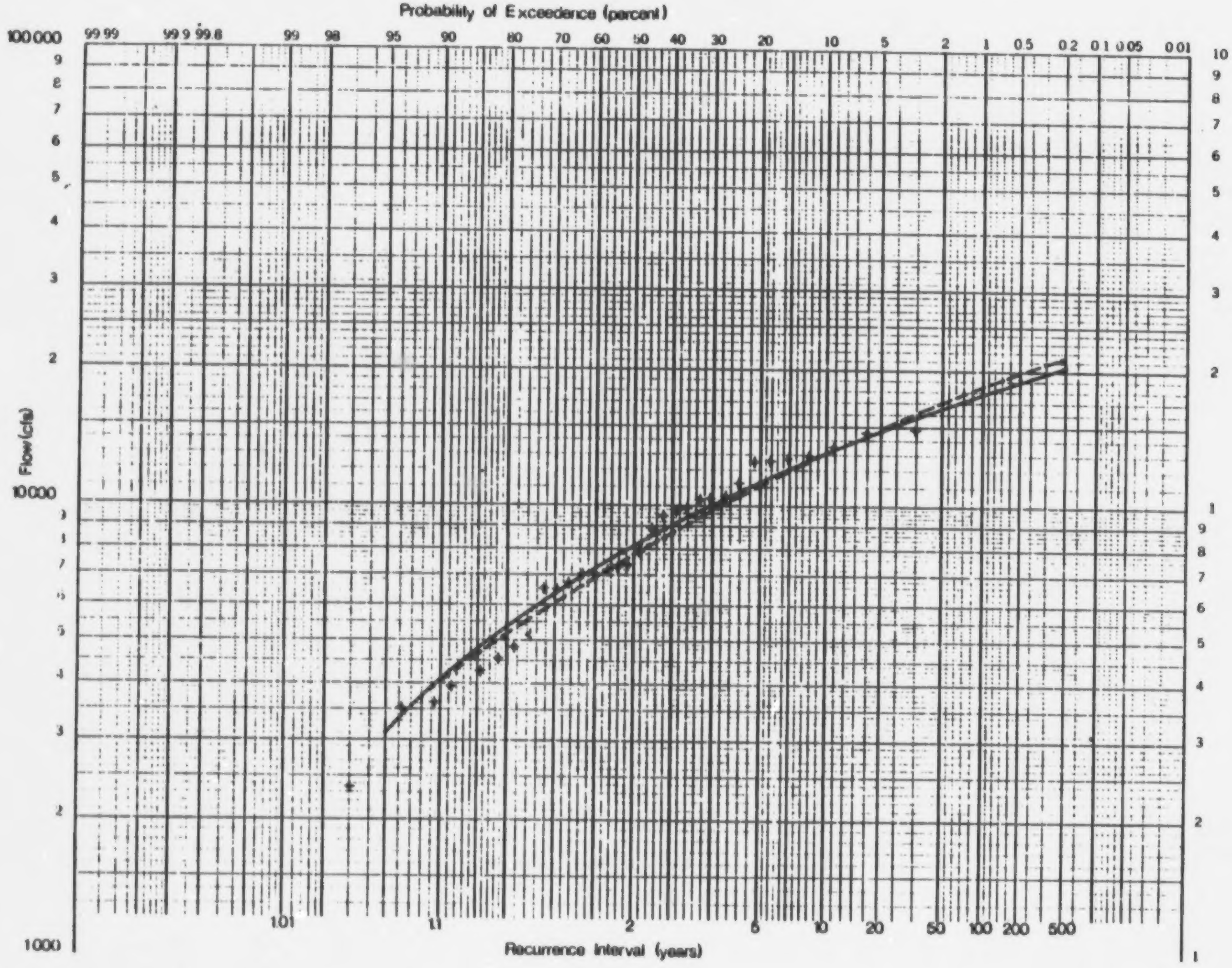


Figure D-7 NITH RIVER AT CANNING
Flood Frequency Analysis
Maximum Instantaneous Flow 1948-1979

—— Three-Parameter Log-Normal Distribution
Parameters Estimated by Maximum Likelihood
--- Log Pearson Type III Distribution
Parameters Estimated by Moments



APPENDIX E

COMPARISON OF FLOOD FREQUENCY FOR THE GRAND RIVER AT GALT 1914-1979 AND 1948-1979.

The flood frequency analysis for the Grand River at Galt was carried out for the period 1948-1979 even though records extend back to 1914. Frequency relationships based on a consistent period of record at all stations were preferred for the development of frequency relationships at ungauged locations.

Flood frequency analysis at Galt for the period of record 1914-1979 is presented in Tables E-1 and E-2.

In reviewing the flow records at Galt from 1914-1947, it was noted that:

a) From 1914 to 1929 only maximum daily flows were available. Maximum instantaneous flows were estimated using the regression equations noted in Figure A-3.

b) Data on reservoir operations at Shand Dam from 1942 to 1947 is sparse. The data is sufficient to provide rough estimates of the daily natural flows. The maximum instantaneous natural flows were estimated using the regression equations noted in Figure A-3. The greater error resulted in several inconsistencies between observed maximum instantaneous flows and the corresponding natural flows that were estimated.

There is some evidence of an increasing trend in flood flows over the last 66 years. The frequency analysis ignores this possible non-stationarity.

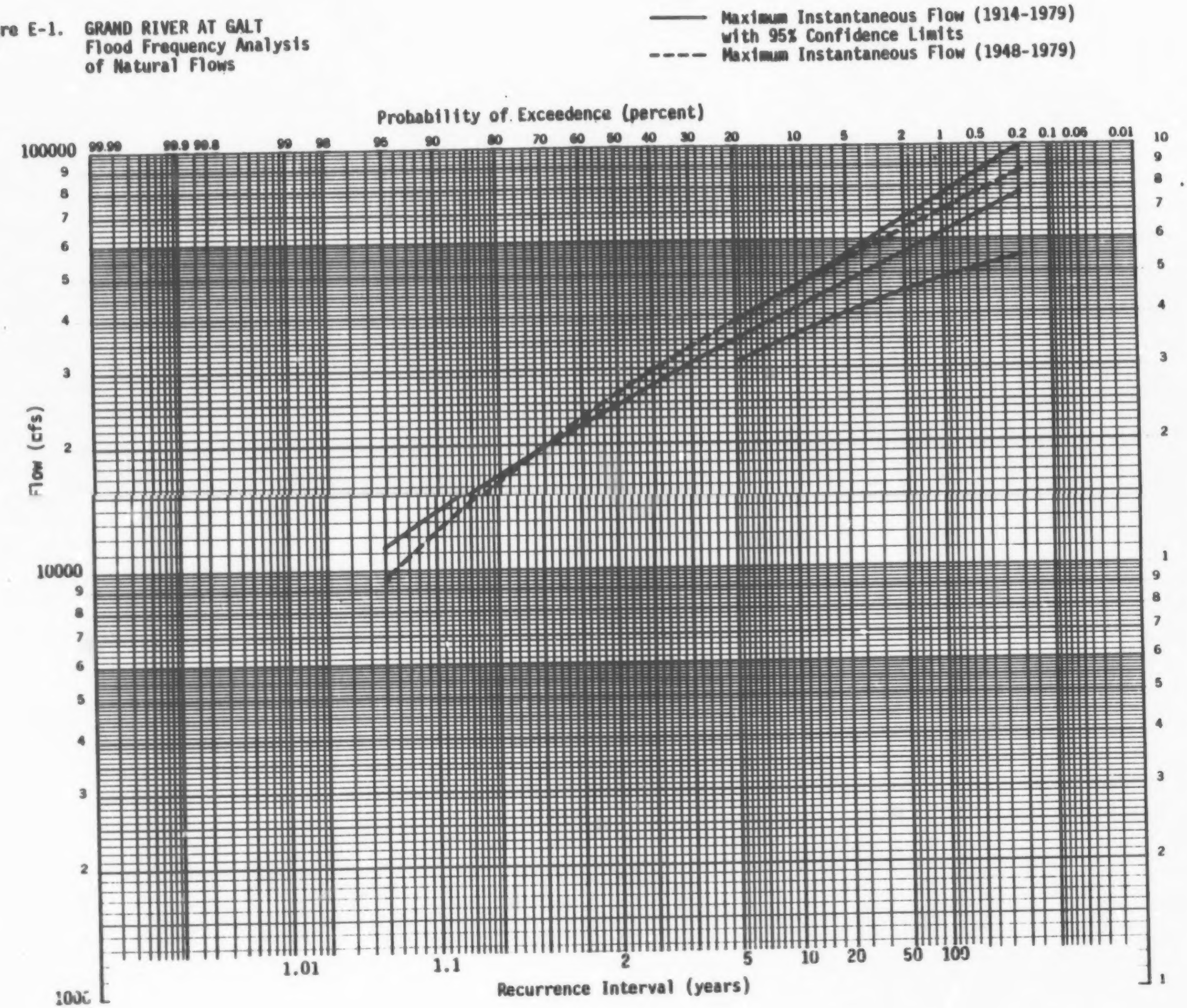
A comparison of the frequency curves for 66 years of record (1914-1979)and the 32 years of record (1948-1979) is shown in Figure E-1. The comparison indicates that the shorter term analysis results in 12 to 14 percent higher flow estimates for the same return period than the longer term analysis. The shorter term frequency curve is within the upper 95% confidence interval of the longer term frequency curve.

TABLE E-1. NATURAL FLOW - GRAND RIVER AT GALT
MAXIMUM INSTANTANEOUS FLOW 1914-1979
IN CUBIC FEET PER SECOND

YEAR	DATA	ORDERED	RANK	PROB.	RET. PERIOD	YEAR	DATA	ORDERED	RANK	PROB.	RET. PERIOD
1914	14500	58700	1	.015	67.000	1947	48500	24400	34	.507	1.971
1915	11300	57600	2	.030	33.500	1948	57600	23900	35	.522	1.914
1916	25500	55200	3	.045	22.333	1949	30100	23800	36	.537	1.861
1917	33100	48500	4	.060	16.750	1950	35900	23400	37	.552	1.811
1918	22500	46300	5	.075	13.400	1951	30300	22700	38	.567	1.763
1919	31400	42800	6	.090	11.167	1952	21400	22700	39	.582	1.718
1920	21000	40400	7	.104	9.571	1953	13600	22500	40	.597	1.675
1921	18200	38700	8	.119	8.375	1954	23900	21600	41	.612	1.634
1922	24600	37300	9	.134	7.444	1955	17200	21400	42	.627	1.595
1923	22700	37000	10	.149	6.700	1956	31100	21400	43	.642	1.558
1924	20800	36000	11	.164	6.091	1957	12200	21000	44	.657	1.523
1925	21600	35900	12	.179	5.583	1958	7210	20800	45	.672	1.489
1926	18800	35900	13	.194	5.154	1959	32900	20200	46	.687	1.457
1927	26700	35000	14	.209	4.786	1960	46300	18800	47	.701	1.426
1928	35000	33200	15	.224	4.467	1961	10400	18300	48	.716	1.396
1929	38700	33100	16	.239	4.188	1962	29700	18200	49	.731	1.367
1930	13000	32900	17	.254	3.941	1963	28200	17900	50	.746	1.340
1931	16000	32400	18	.269	3.722	1964	15000	17200	51	.761	1.314
1932	31700	31700	19	.284	3.526	1965	26700	17200	52	.776	1.288
1933	17200	31400	20	.299	3.350	1966	9380	16300	53	.791	1.264
1934	18300	31100	21	.313	3.190	1967	37000	16200	54	.806	1.241
1935	20200	30300	22	.328	3.045	1968	14600	16000	55	.821	1.218
1936	16300	30100	23	.343	2.913	1969	23800	16000	56	.836	1.196
1937	16000	29700	24	.358	2.792	1970	17900	15000	57	.851	1.175
1938	23400	28200	25	.373	2.680	1971	21400	14600	58	.866	1.155
1939	27000	28200	26	.388	2.577	1972	32400	14500	59	.881	1.136
1940	24400	27000	27	.403	2.481	1973	28200	13600	60	.896	1.117
1941	16200	26700	28	.418	2.393	1974	55200	13000	61	.910	1.098
1942	35900	26700	29	.433	2.310	1975	58700	12200	62	.925	1.081
1943	37300	26600	30	.448	2.233	1976	42800	11300	63	.940	1.063
1944	22700	26100	31	.463	2.161	1977	40400	10400	64	.955	1.047
1945	33200	25500	32	.478	2.094	1978	26600	9380	65	.970	1.031
1946	26100	24600	33	.493	2.030	1979	36000	7210	66	.985	1.015

```
TABLE E-2.   NATURAL FLOW  GRAND RIVER AT GALT
            MAXIMUM INSTANTANEOUS FLOW 1914-1979
                  IN CUBIC FEET PER SECOND

              SAMPLE STATISTICS

        MEAN =      26423         S.D. =   11376.8        C.S. =      .0784         C.K. =    3.9156

              SAMPLE STATISTICS (LOGS)

        MEAN =   10.0901          S.D. =      .4412       C.S. =    ¯.2872          C.K. =    3.1633

        SAMPLE MIN =    7210     SAMPLE MAX =   58700     N =  66

        PARAMETERS FOR GUMBEL I     A =  .000111     U =     21244

        PARAMETERS FOR LOGNORMAL     M = 10.0901     S =  .4412

        PARAMETERS FOR THREE PARAMETER LOGNORMAL     A =    ¯5813     M = 10.3216     S =  .3481

              STATISTICS OF LOG(X-A)

        MEAN =   10.3216          S.D. =      .3481       C.S. =    ¯.0335          C.K. =    2.9416

        PARAMETERS FOR LOG PEARSON III BY MOMENTS     A= ¯.0633 B=   48.5069 LOG(M)=13.1626     M =      520549

        PARAMETERS FOR LOG PEARSON III BY MAXIMUM LIKELIHOOD     A = ¯.0710     B =   38.1182   LOG(M) =12.7965     M =      360943

        DISTRIBUTION STATISTICS     MEAN = 10.0901     S.D =  .4383     C.S. = ¯.3239



                  GUMBEL I             LOGNORMAL         THREE PARAMETER                LOG PEARSON III
                                                           LOGNORMAL        MAXIMUM LIKELIHOOD           MOMENTS
   RETURN     FLOOD   ST.ERROR  FLOOD   ST. ERROR  FLOOD   ST. ERROR  FLOOD   ST. ERROR  FLOOD   ST. ERROR
   PERIOD    ESTIMATE  PERCENT  ESTIMATE  PERCENT  ESTIMATE  PERCENT  ESTIMATE  PERCENT  ESTIMATE  PERCENT

    1.005      6280                9270                8480                8560                8580
    1.050     11300               11700               11300               11300               11300
    1.250     17000               16600               16800               16800               16800
    2.000     24500               24100               24600               24700               24600
    5.000     34700      5.74     34900      6.32     34900      5.82     35000      5.75     35100     10.30
   10.000     41400      6.16     42400      7.33     41700      6.77     41500      6.33     41800     12.70
   20.000     47900      6.52     49800      8.33     48000      8.06     47500      7.41     48000     16.10
   50.000     56300      6.91     59600      9.57     56200      9.94     54800      9.38     55600     21.60
  100.000     62500      7.14     67300     10.50     62500     11.40     60200     11.10     61300     26.20
  200.000     68800      7.34     75100     11.30     68700     12.90     65300     13.00     66700     31.10
  500.000     77000      7.56     85900     12.30     77000     14.80     71900     15.60     73800     37.90
```

Figure E-1. GRAND RIVER AT GALT
Flood Frequency Analysis
of Natural Flows



APPENDIX F

EFFECT OF HURRICANE HAZEL ON THE FLOOD FREQUENCY CURVE

The flood frequency analysis was carried out on maximum instantaneous flows generated during the spring. However, it is generally agreed that the upper end of the frequency curve will be dominated by events generated by tropical storms. Only one tropical storm has produced an annual maximum flow: Hurricane Hazel, October 1954. Figures D-1 to D-5 show the effect of including this event on the frequency curves.

Figure F-1. GRAND RIVER AT GALT
Flood Frequency Analysis
Maximum Instantaneous Flow 1948-1979

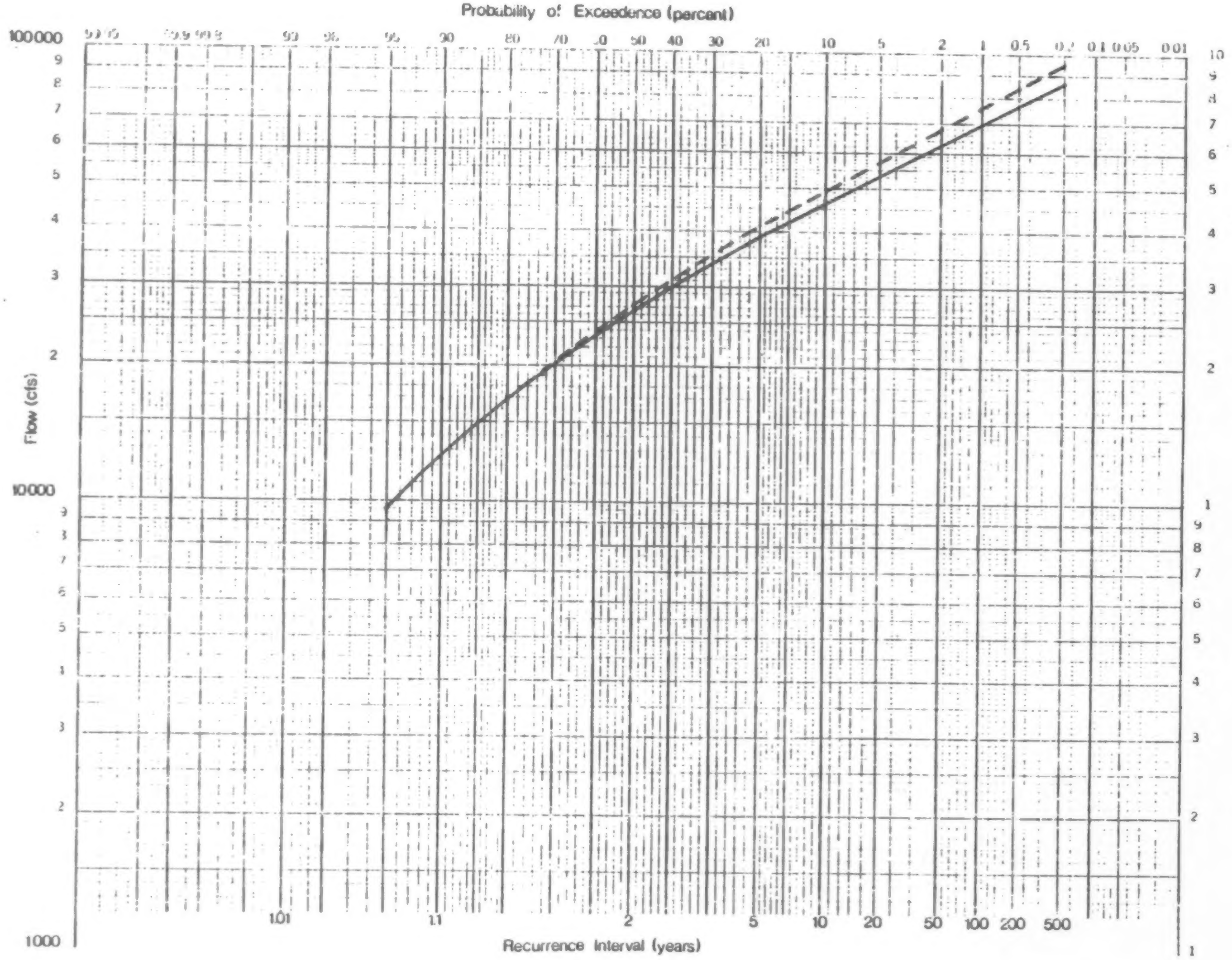


Figure F-2. GRAND RIVER AT BRANTFORD
Flood Frequency Analysis
Maximum Instantaneous Flow 1948-1979

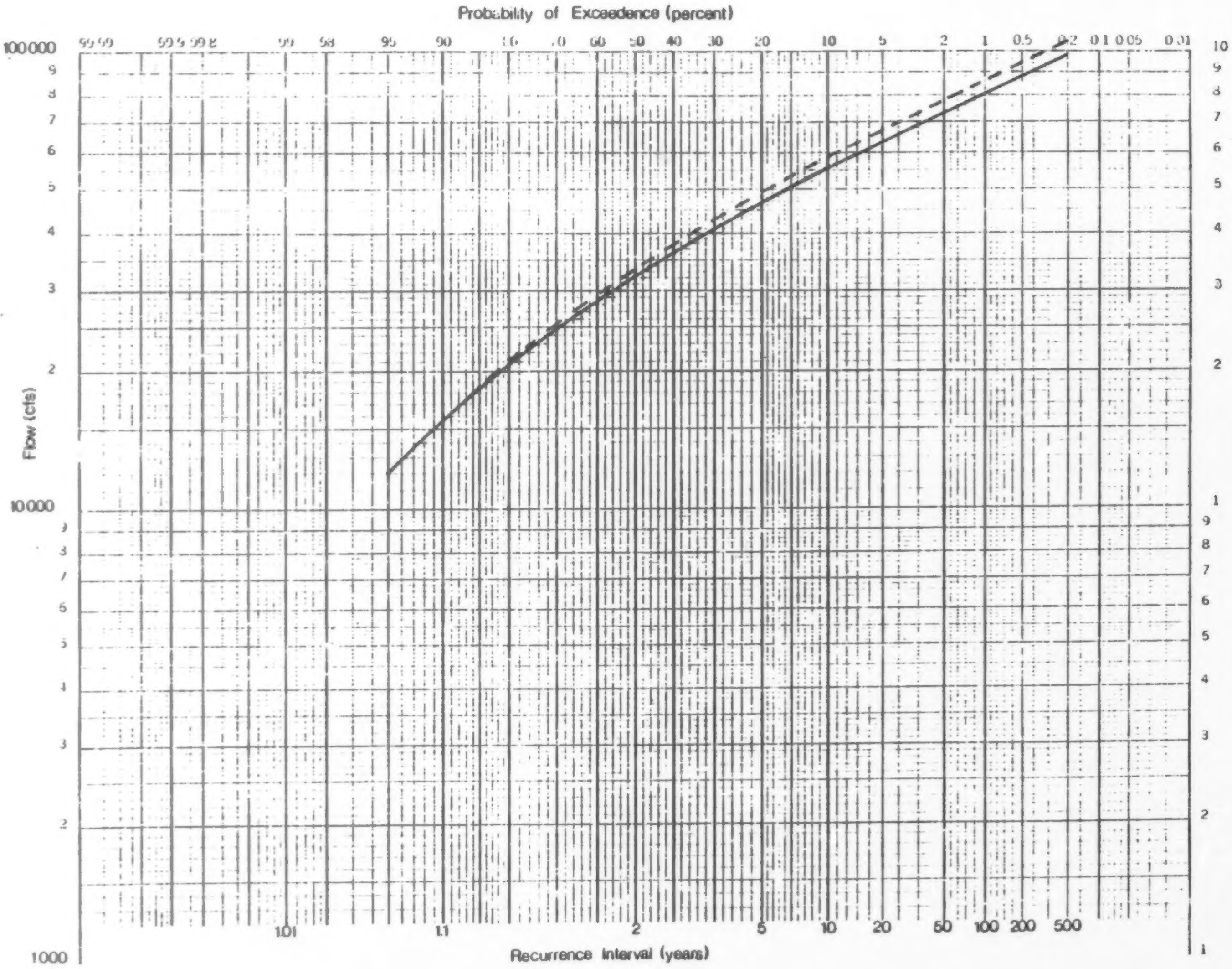


Figure F-3. SPEED RIVER BELOW GUELPH
Flood Frequency Analysis
Maximum Instantaneous Flow 1950-1979

——— Hurricane Hazel not included.
– – – Hurricane Hazel included.

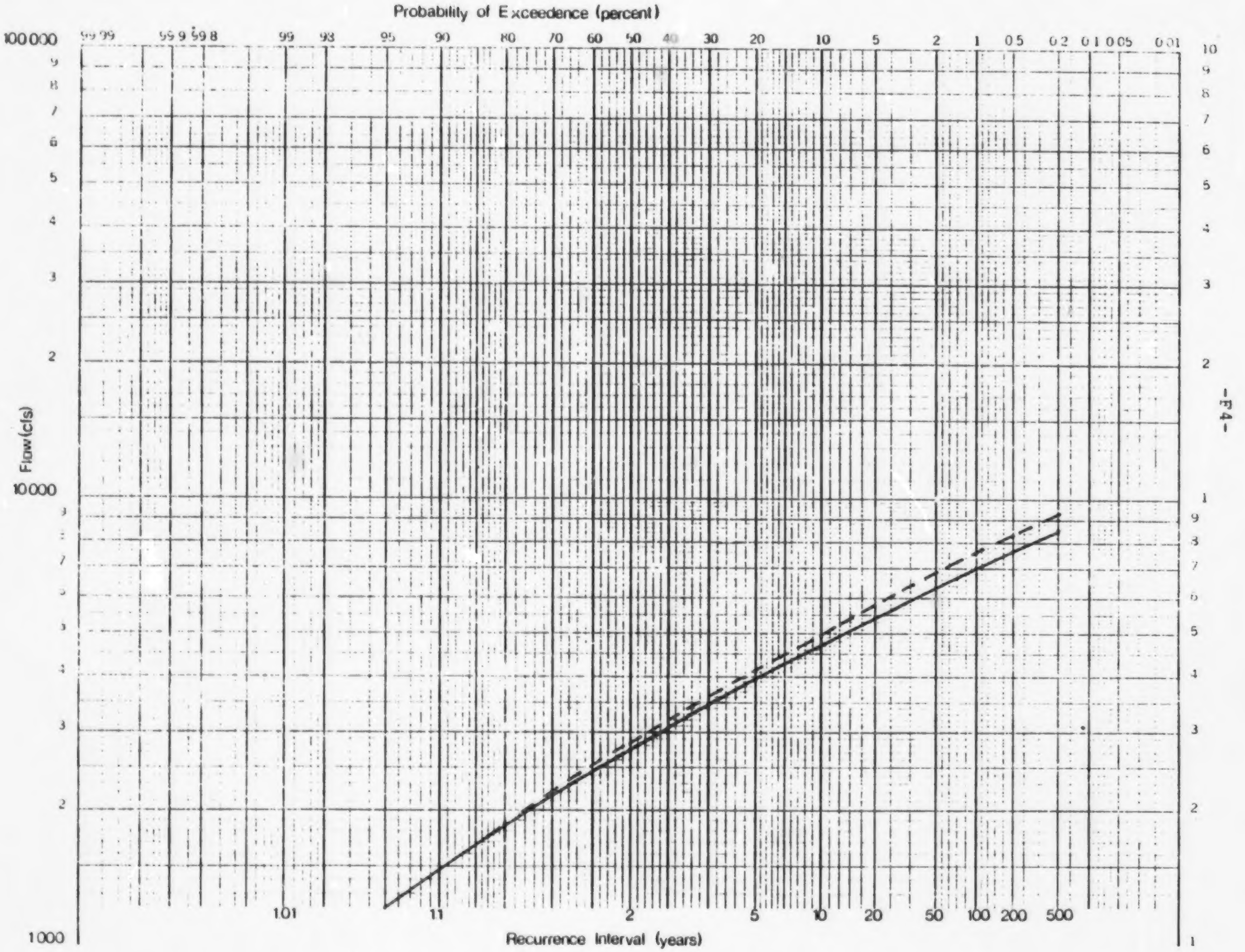


Figure F-4. NITH RIVER AT NEW HAMBURG
Flood Frequency Analysis
Maximum Instantaneous Flow 1951-1979

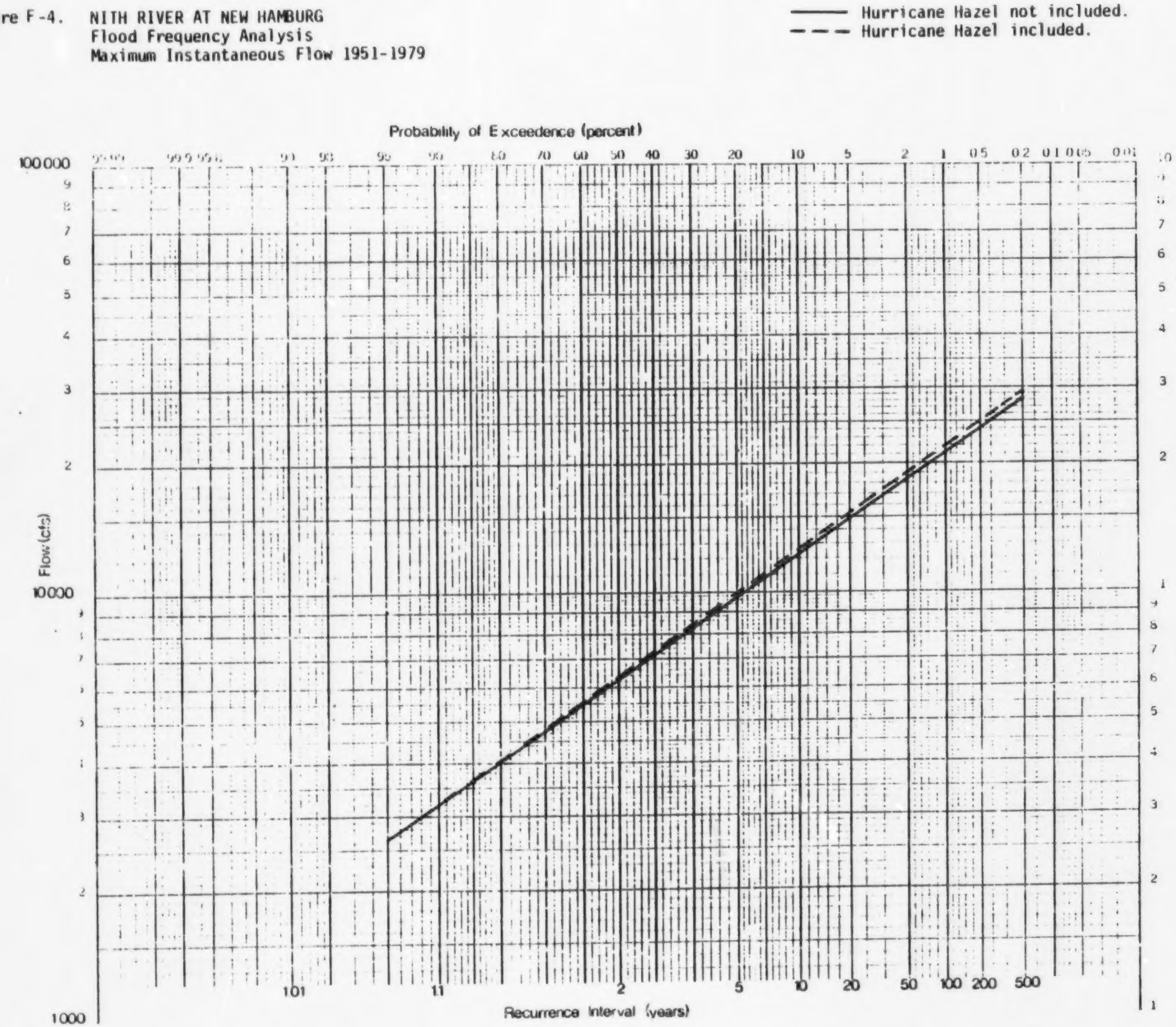
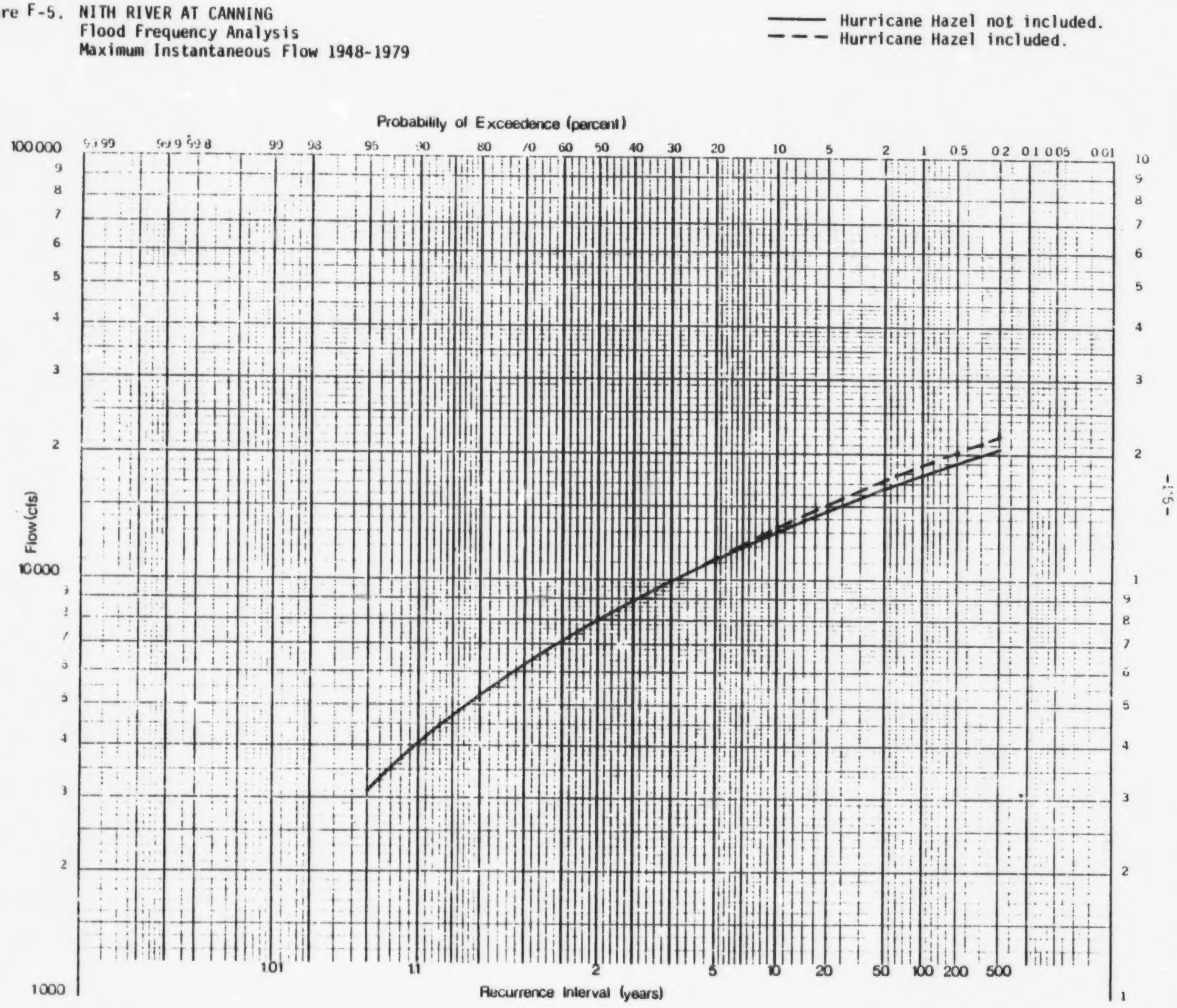


Figure F-5. NITH RIVER AT CANNING
Flood Frequency Analysis
Maximum Instantaneous Flow 1948-1979



APPENDIX G

COMPARISON OF FLOOD FREQUENCY WITH REGIONAL FLOOD FREQUENCY ANALYSIS COMPLETED BY ENVIRONMENT CANADA

"Magnitude and Frequency of Flood in Southern Ontario", B.P. Sangal and R.W. Kallio, Technical Bulletin Series No. 99, Environment Canada, 1977 details a regional flood frequency analysis for Southern Ontario.

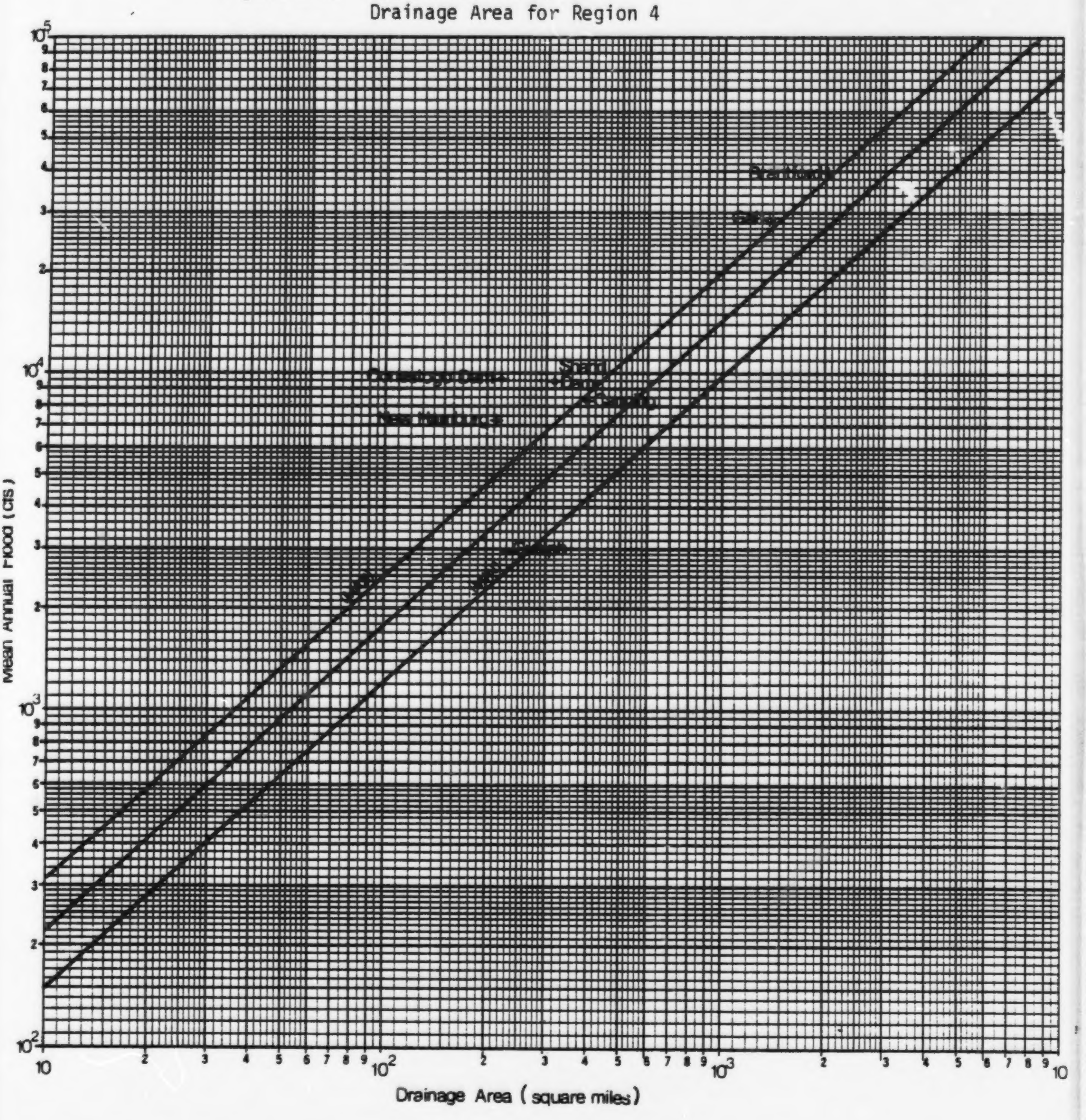
Southern Ontario was divided into nine hydrologic regions with roughly similar soils and physiographic characteristics. For each region a graphical relationship between the mean annual flood and drainage area was determined. Dimensionless frequency curves (ratio to mean annaul flood) based on the three-parameter log-normal distribution were derived for each region, giving the ratio of flood estimate probabilities.

These curves for Region 4, Maitland-Thames-Grand System, have been reproduced as Figures E-1 to 8. The corresponding results of this study are shown on the curves.

In the case of mean annual flood versus drainage area, the higher flows resulting from the clay loam soils the north-western portion of the Grand River basin tend toward the maximum values of the regional curve; the lower flows from the gravel soils of the eastern Grand River basin tend toward the minimum values of the regional curve.

In the case of the dimensionless frequency curves, the actual curve at all locations lies at or below the regional curve.

Figure G-1. Mean Annual Flood vs.
Drainage Area for Region 4



+ actual values for stations included in this study

Figure G-2. Regional Frequency Curve for Region 4

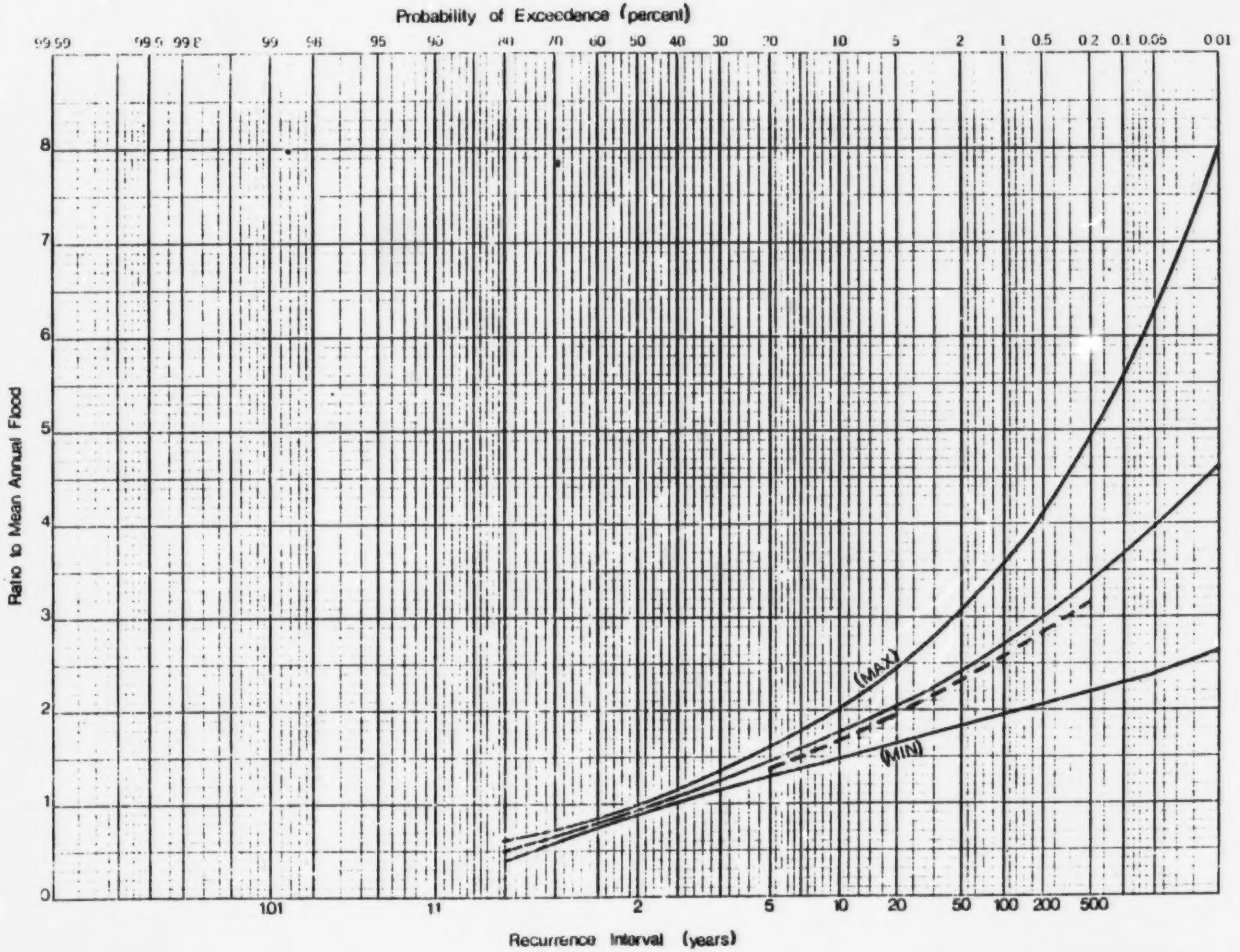


Figure G-3. Regional Frequency Curve for Region 4

--- CONESTOGO RIVER AT CONESTOGO DAM

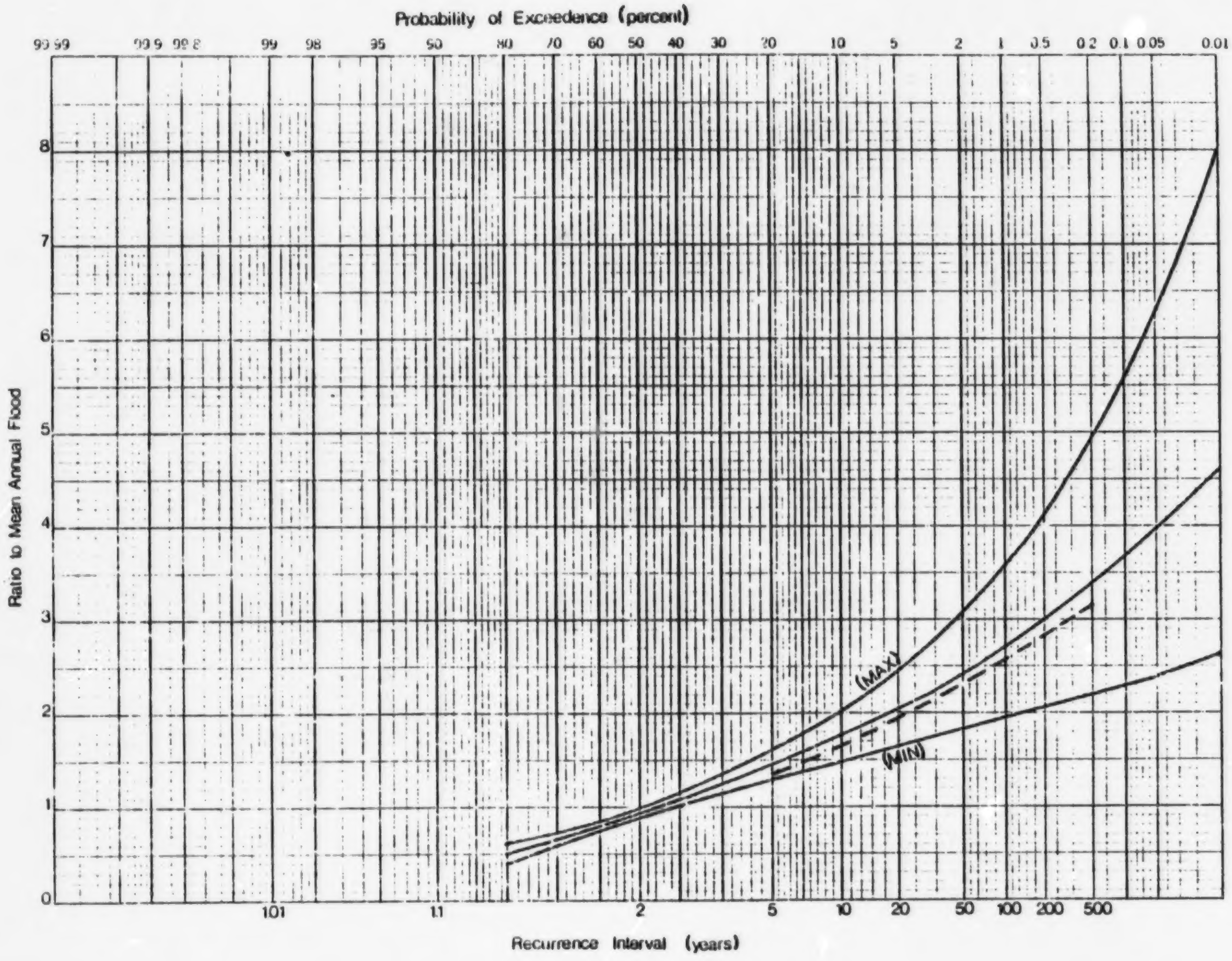


Figure G-4. Regional Frequency Curve for Region 4

— — — GRAND RIVER AT GALT

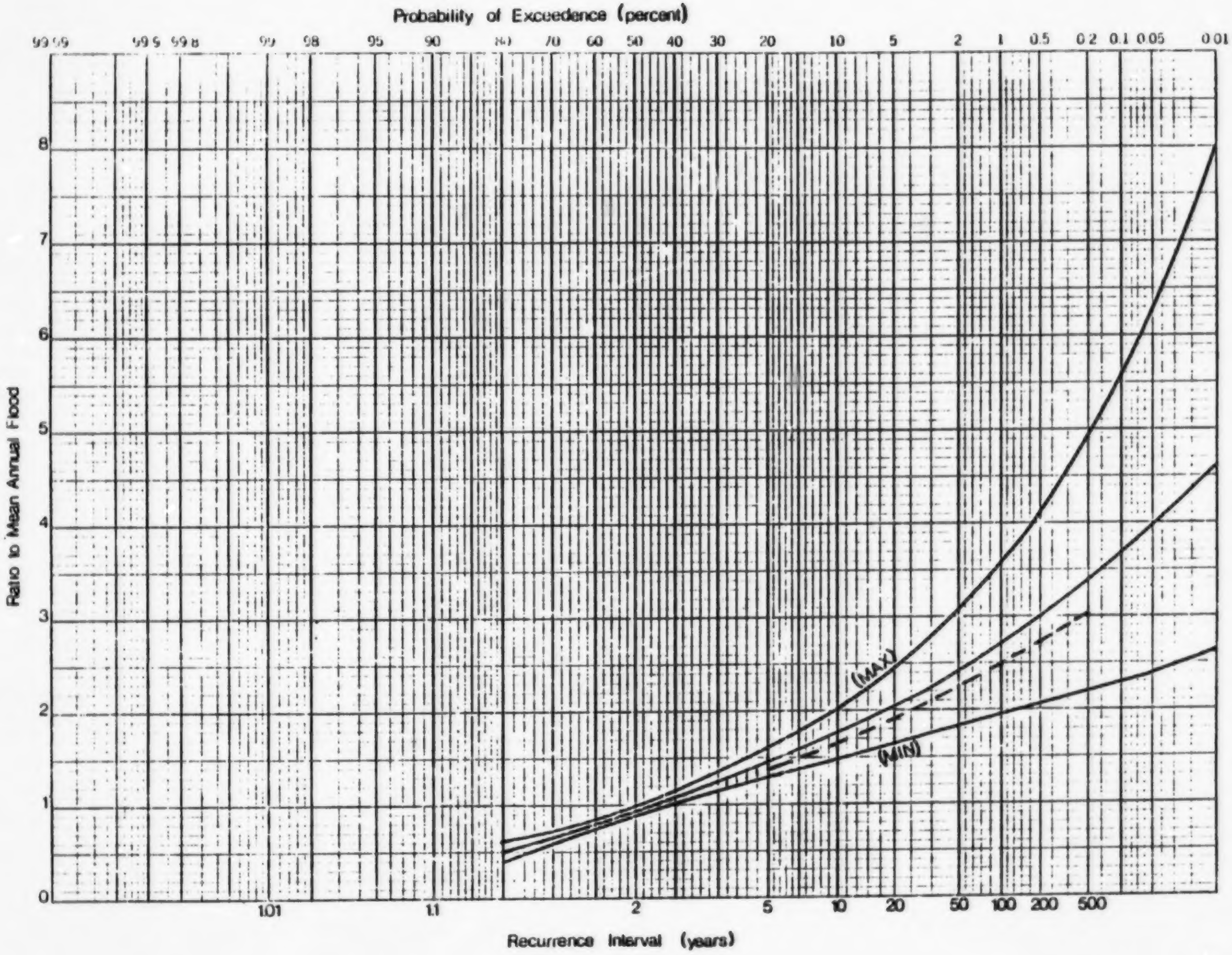


Figure G-5. Regional Frequency Curve for Region 4

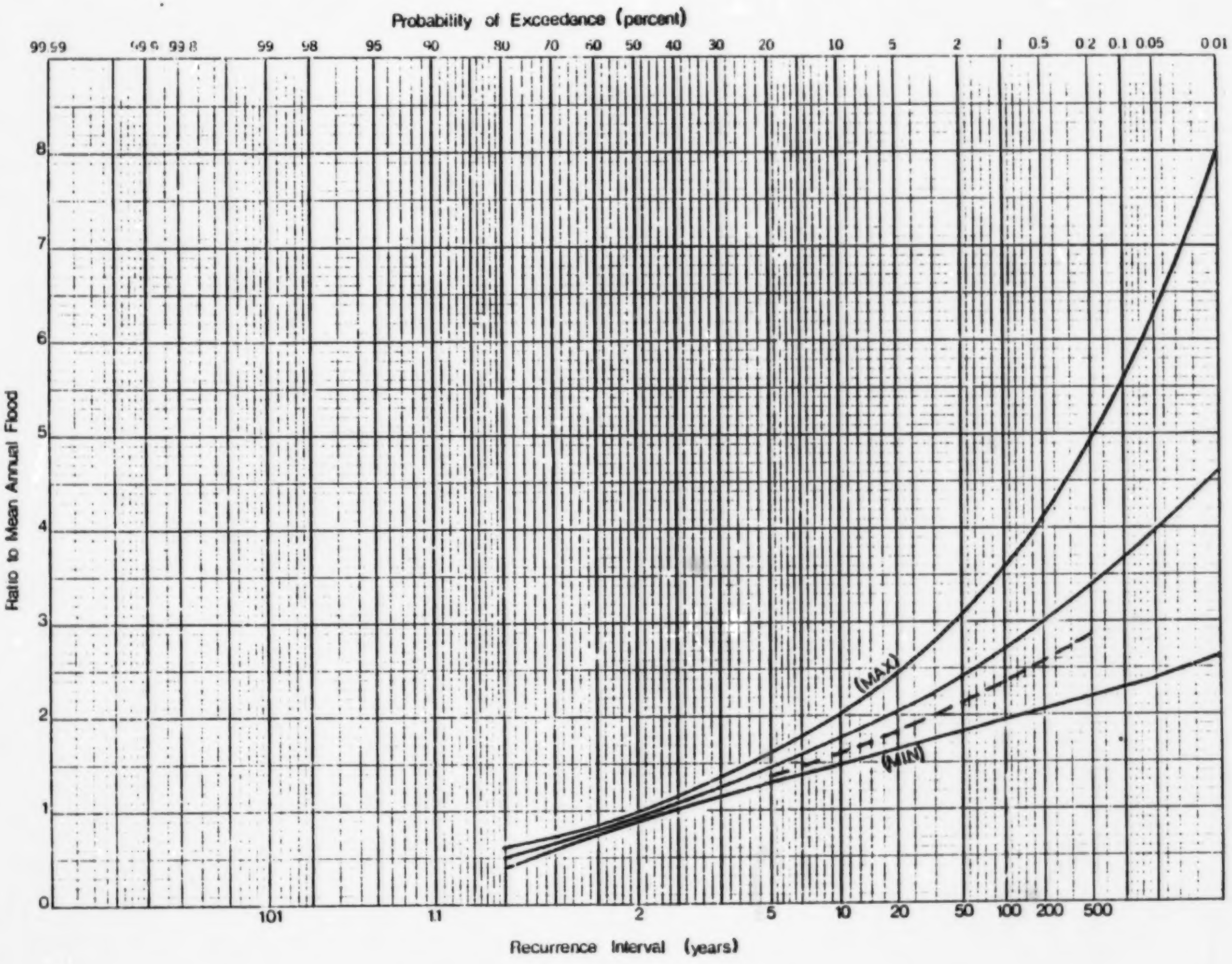


Figure G-6. Regional Frequency Curve for Region 4

--- SPEED RIVER BELOW GUELPH

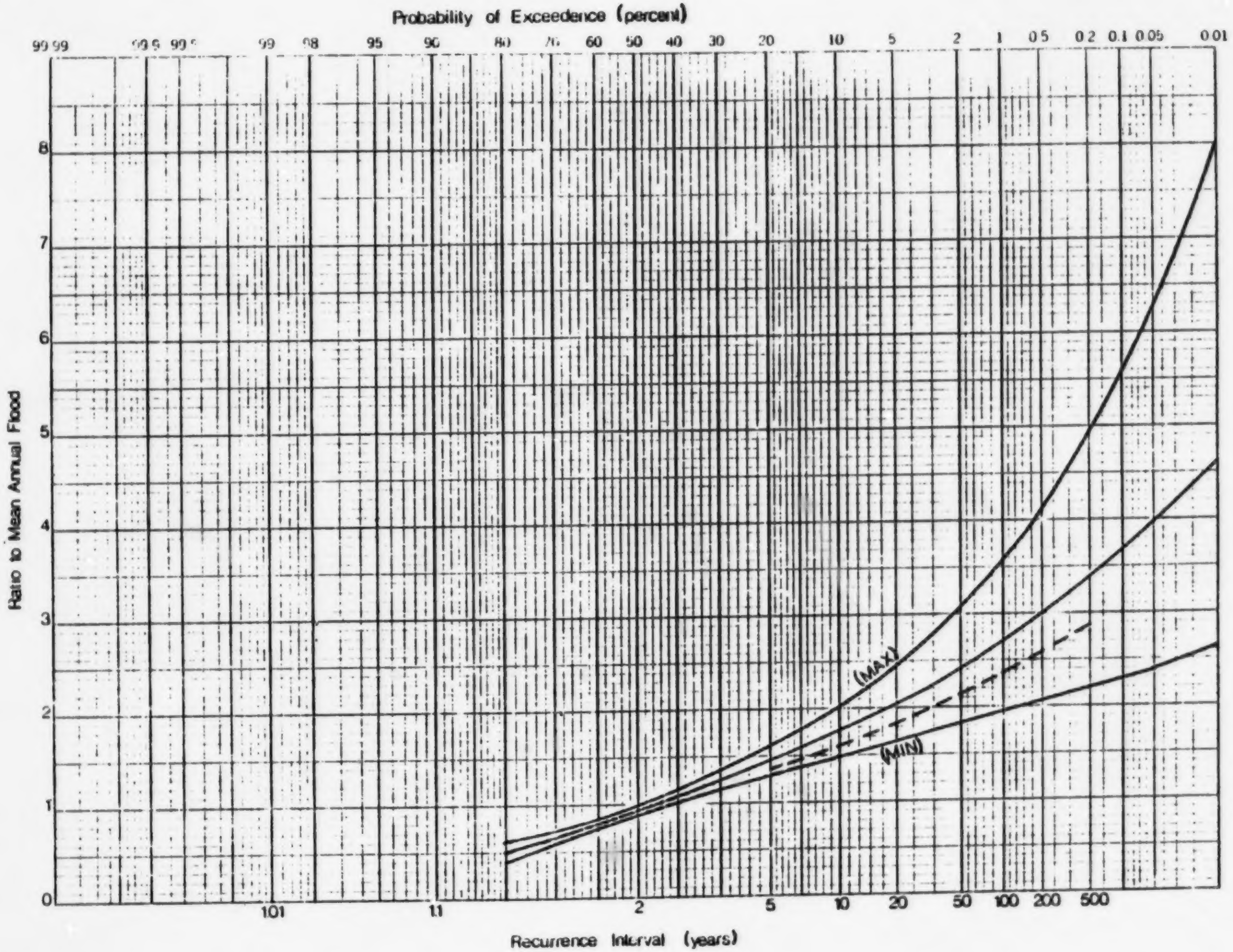


Figure G-7. Regional Frequency Curve for Region 4

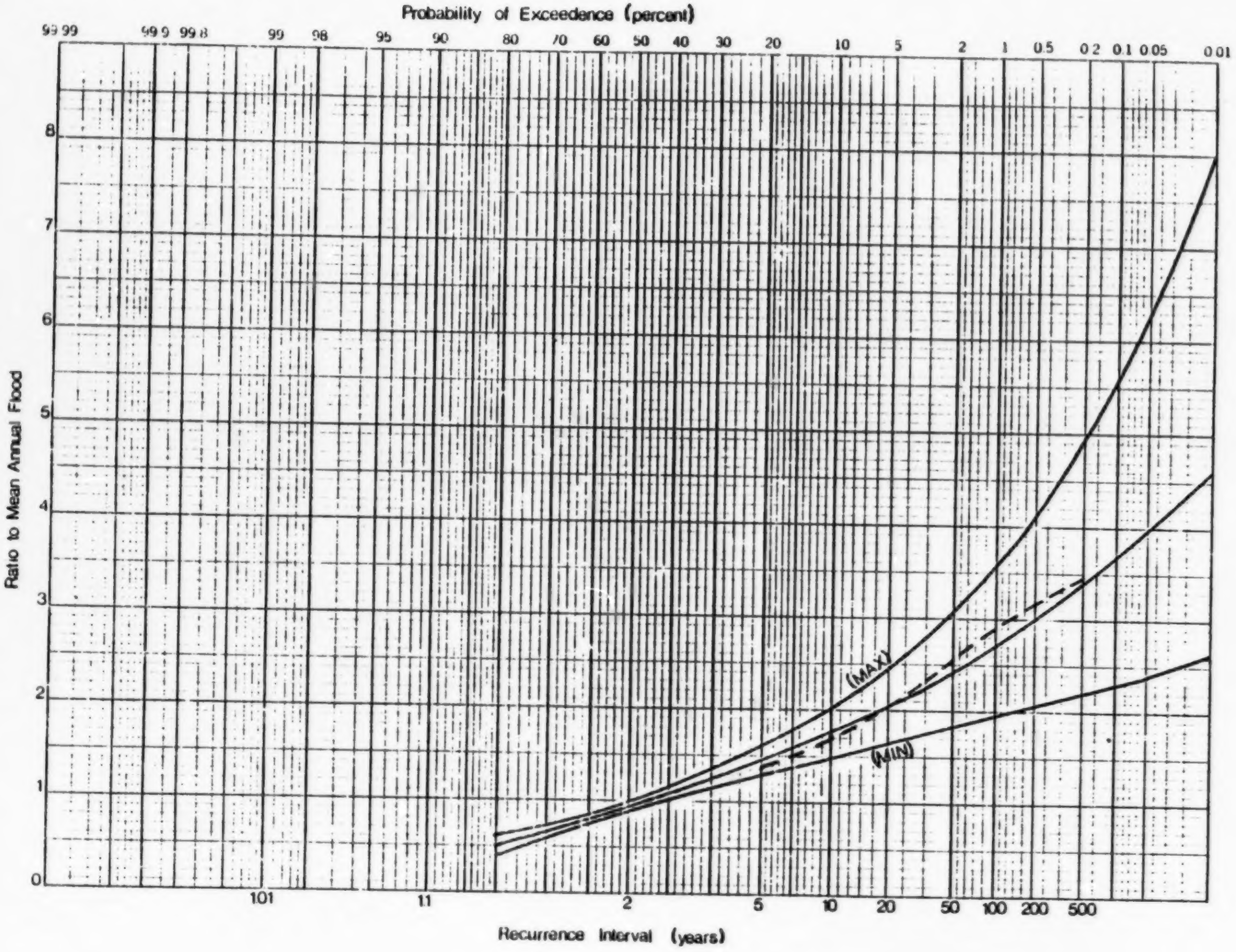
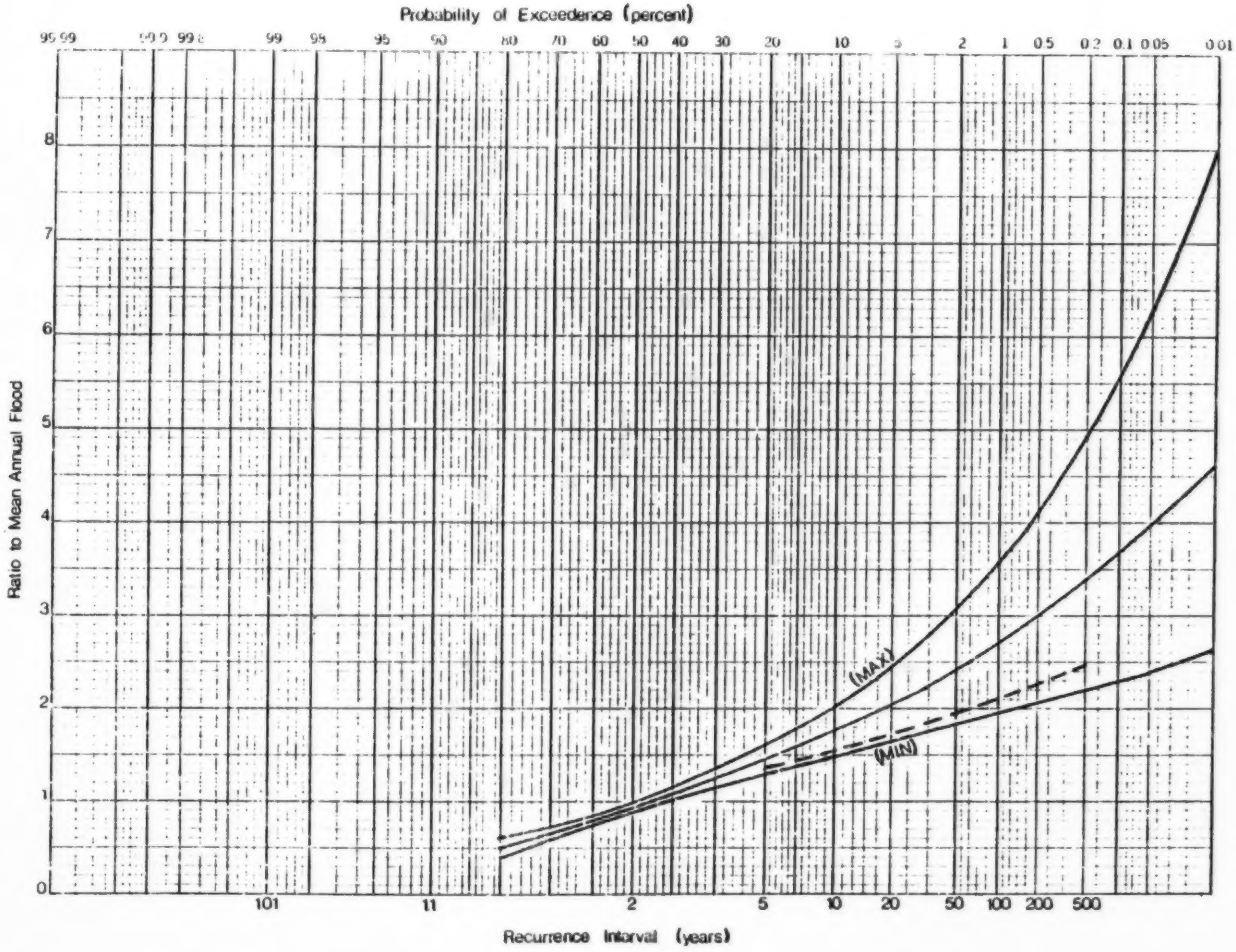


Figure G-8. Regional Frequency Curve for Region 4


--- NITH RIVER AT CANNING
Probability of Exceedence (percent)
99.99
99.9
99.8
99
98
95
90
80
70
60
50
40
30
20
10
5
2
1
0.5
0.2
0.1
0.05
0.01
Ratio to Mean Annual Flood
0
1
2
3
4
5
6
7
8
(MAX)
(MIN)
101
11
2
5
10
20
50
100
200
500
Recurrence Interval (years)